MATH 417: Numerical Analysis

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Homework assignment 10 – due 4/26/2007

Problem 1 (Numerical integration.) Consider the problem of finding the numerical value of the integral

$$\int_0^1 \arctan x \, dx.$$

The exact value of this expression is $\frac{\pi}{4} - \frac{\ln 2}{2} = 0.43882\ldots$.

Evaluate above integral by writing programs that use

(a) the trapezoidal rule,

(b) the Simpson rule.

Split up the integration interval $[0, 1]$ into successively smaller sub-intervals of length $h = 1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{128}$ and apply the two quadrature rules above to each subinterval. Compute the approximated value of the integral and the error. Determine the convergence order from this data. **(4 points)**

Problem 2 (Integration of an implicit function). Let $f(x)$ be defined as in last week's homework, i.e. $f(x)$ is that value y for which $ye^y = x$. Compute

$$\int_0^{10} f(x) \, dx$$

using the trapezoidal rule for step sizes $h = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{32}$. Determine the order of convergence. **(4 points)**

Problem 3 (Numerical solution of a ODE). Consider the following scalar ordinary differential equation (ODE):

$$x'(t) = \frac{1}{2}x(t), \qquad x(0) = 1.$$

The solution of this equation is $x(t) = e^{\frac{1}{2}t}$. Compute approximations to $x(4)$ using the

- first order Taylor expansion method,
- second order Taylor expansion method,

- implicit Euler method,

each with step sizes $h = 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{32}$. Compute their respective errors $e = |x_N - x(4)|$ where x_N is the approximation to $x(4)$ at the end of the last time step, and compute the convergence rates. Compare the accuracy of all these methods for the same step size h. **(7 points)**