

MATH 417: Numerical Analysis

Instructor: Prof. Wolfgang Bangerth
bangerth@math.tamu.edu,

Teaching Assistants: Dukjin Nam
dnam@math.tamu.edu

Homework assignment 2 – due 2/8/2007

Problem 1 (Bisection method). A ball is dropped from a tower. During its fall it is subject to the forces of gravity and air resistance. Thus, the height h in ft as a function of time t in s is given by the formula

$$h(t) = h_0 - \frac{mg}{k}t + \frac{m^2g}{k^2} \left(1 - e^{-\frac{kt}{m}}\right).$$

Here, $h_0 = 300ft$ is the height of the tower, $m = 1/2lb$ the mass of the ball, $g = 32.17ft/s^2$ is the gravitation of earth and $k = 1/4lb\,s/ft$ is the air resistance coefficient.

Use the bisection method (on paper, with pen and calculator) to find out when the ball hits the ground with an accuracy of 0.1 seconds. **(6 points)**

Problem 2 (Newton's method). For certain functions, Newton's method will always converge in a single step, no matter where we start. What functions are these, and why is a single step enough? (Hint: Think about the graphical interpretation of Newton's method, and when it will produce a new iteration that falls exactly onto the true root of the function.) **(2 points)**

Problem 3 (Newton's method). For functions $f(x)$ of one variable x , Newton's method almost always converges very quickly (in a matter of a few iterations). However, almost always is not always, and we can find examples where Newton's method converges rather slowly.

Write a program to find the zero $x = 1$ of the function

$$f(x) = x^{25} - 1$$

that uses Newton's method and starts at $x_0 = 20$.

- How many iterations do you need to achieve an accuracy of 10^{-8} ?
- You will observe very slow convergence. Can you explain from the formulas that express the error e_n as a function of e_{n-1} why convergence is so slow?
- Does the method still converge of second order, i.e. is the relationship between e_n and e_{n+1} derived in class true also for this problem?
- What answers do you get to the questions in (a)–(c) if you apply the same program to the function $f(x) = x^3 - 1$ instead, again starting from $x_0 = 20$? **(8 points)**

Problem 4 (Fixed-point iteration). Let a be a positive number and

$$g(x) = 2x - ax^2.$$

- (a) Show that the limit of this fixed-point iteration is either 0 or $1/a$.
- (b) This iteration gives us a means to compute the reciprocal value of a using only multiplication and subtraction. Find (analytically) an interval containing $1/a$ in which this iteration converges to $1/a$.

(4 points)