MATH 417: Numerical Analysis

Instructors:	Prof. Wolfgang Bangerth, Prof. Guido Kanschat bangerth@math.tamu.edu, kanschat@math.tamu.edu
Teaching Assistants:	Seungil Kim, Yan Li sgkim@math.tamu.edu, yli@math.tamu.edu

Homework assignment 8 – due 11/2/06 and 11/6/06

Problem 1 (Condition numbers). Calculate the condition numbers $\kappa(A) = \|A\|\,\|A^{-1}\|$ with respect to the l_1, l_∞ and l_2 norms for the matrix

$$A = \begin{pmatrix} 1 & 1.001 \\ 0.999 & 1 \end{pmatrix}.$$

(5 points)

Problem 2 (Error propagation). With the matrix from Problem 1, consider the solutions $x, \tilde{x}$ of the following linear systems:

$$Ax = b, \qquad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$A\tilde{x} = \tilde{b}, \qquad \tilde{b} = \begin{pmatrix} 1 \\ 1.001 \end{pmatrix}.$$

(Imagine the former to be the exact right hand side, and the latter to be one that is contaminated by measurement uncertainty, statistical error, etc.)

Solve for x and $\tilde{x}$. Calculate the relative difference in the right hand side $\epsilon_r = \|b - \tilde{b}\|/\|b\|$ and the relative error $e_r = \|x - \tilde{x}\|/\|x\|$ in the solution, each for both the l_2 and the l_∞ norm.

Using your result from Problem 1, do ϵ_r and e_r satisfy the estimates discussed in class? **(5 points)**

Problem 3 (Lagrange interpolation).

(a) Compute the Lagrange interpolation polynomials $L_{4,k}, k = 0 \ldots 3$, for the points $x_0 = 1$, $x_1 = 2$, $x_2 = 1.5$ and $x_3 = 1.6$.

(b) Calculate the interpolating polynomial for the data set where $y_k = \log x_k$ at the four points x_k. Write the polynomial in the form $a_3x^3 + a_2x^2 + a_1x + a_0$.

(c) The polynomial calculated in (b) by construction interpolates the function $f(x) = \log x$. Compute the maximal error on the interval $[1, 2]$.

(6 points)