MATH 417: Numerical Analysis

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Homework assignment 7 – due 10/26/06 and 10/30/06

Problem 1 (Jacobi iteration). Solve problems 7.3.1 a) and b) of the book (using paper and pencil). **(3 points)**

Problem 2 (Gauss-Seidel iteration). Solve problems 7.3.3 of the book (using paper and pencil) for parts a) and b). **(3 points)**

Problem 3 (Convergence of Richardson iteration). Richardson iteration is another variant of the methods presented. It can be written in the form

$$x^{k+1} = x^k - \omega\big(Ax^k - b\big),$$

where $\omega > 0$ is called the damping factor. Follow these steps to prove that this method yields a contraction with respect to the norm $\|.\|_2$ if A is a symmetric positive definite matrix and $0 < \omega < 1/\lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of A.

a) State what the operator $T = \mathbf{1} - BA$ of the iteration is, by identifying what B is for the iteration above.

b) Rewrite the product Tx in terms of the eigenvalues and eigenvectors of A.

c) Use this decomposition to estimate the spectral radius $\rho(T^T T)$.

c) Draw your conclusions concerning the contraction property of T, i.e. show whether or not $\|T\|_2 = \|\mathbf{1} - BA\|_2 < 1$ for the given choice of ω.

(4 points)

Problem 4 (Gauss-Seidel iteration). Repeat Problem 3 of Homework 6 (the Jacobi iteration), but use the Gauss-Seidel iteration instead to compute the vectors $x^{(k)}$. Generate the same plots as before. Compare your results with the previous results. **(4 points)**