MATH 417: Numerical Analysis

Instructors:	Prof. Wolfgang Bangerth, Prof. Guido Kanschat bangerth@math.tamu.edu, kanschat@math.tamu.edu
Teaching Assistants:	Seungil Kim, Yan Li sgkim@math.tamu.edu, yli@math.tamu.edu

Homework assignment 3 – due 9/21/06 and 9/22/06

Problem 1 (Newton's method). For certain functions, Newton's method will always converge in a single step, no matter where we start. What functions are these, and why is a single step enough? (Hint: think about the graphical interpretation of Newton's method, and when it will produce a new iteration that falls exactly onto the true root of the function.) **(2 points)**

Problem 2 (Newton's method). For functions $f(x)$ of one variable x, Newton's method almost always converges very quickly (in a matter of a few iterations). However, almost always is not always, and we can find examples where Newton's method converges rather slowly.

Write a program to find the zero $x = 1$ of the function

$$f(x) = x^{25} - 1$$

that uses Newton's method and starts at $x_0 = 20$.

(a) How many iterations do you need to achieve an accuracy of 10^{-8}?

(b) You will observe very slow convergence. Can you explain from the formulas that express the error e_n as a function of e_{n-1} why convergence is so slow?

(c) Does the method still converge of second order? **(6 points)**

Problem 3 (Newton's method for convex functions). Prove that Newton's method converges from any point p_0 with $f'(p_0) \neq 0$ to a root of the function f, provided that f has at least one zero, and is globally convex, i.e. $f''(x) > 0$ for all x.

(4 points)

Problem 4 (Convergence order). Determine the order of convergence and the asymptotic error constant for the following sequences:

(a) $a_n = 5.0625, 2.25, 1, \frac{4}{9}, \frac{16}{81}$

(b) $b_n = 2.718, 2.175, 1.740, 1.392, 1.113, 0.8907$

(c) $c_n = 0.318, 0.180, 0.0761, 0.021, 3.04 \cdot 10^{-3}, 1.68 \cdot 10^{-4}, 2.17 \cdot 10^{-6}$. **(3 points)**