MATH 412: Theory of Partial Differential Equations

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Homework assignment 7 – due Thursday 10/26/2006

Problem 1 (Fourier series). Derive the Fourier series on $[-\pi, \pi]$ of the function $f(x) = x$. From this series, derive the Fourier series of $F(x) = x^2/2$ without using the formulas $\frac{1}{L}\int_{-L}^{L} F(x) \cos nx \, dx$ (and similar for the sine terms) to compute the coefficients A_0, A_n, B_n of the second series. **(3 points)**

Problem 2 (Wave equation). The wave equation with constant coefficients and zero right hand side reads in one space dimension

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0,$$

where c is the so-called wave speed. Show that if u has the form $u(x,t) = f(x - ct)$ for an arbitrary function $f(s)$, then $u(x,t)$ is a solution of the wave equation. Show that the same is true for $u(x,t) = g(x + ct)$. How about $u(x,t) = \alpha f(x - ct) + \beta g(x + ct)$? **(4 points)**

Problem 3 (Wave equation). Solve problem 4.2.1 in the book. **(3 points)**