MATH 412: Theory of Partial Differential Equations

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Homework assignment 1 – due Thursday 9/7/2006

Problem 1 (Equilibrium temperatures). Solve exercise 1.4.1 in the book, parts b and g. **(2 points)**

Problem 2 (Binary materials). Solve exercise 1.4.3 in the book. **(3 points)**

Problem 3 (Heat energy). Solve exercise 1.4.10 in the book. **(2 points)**

Problem 4 (Not quite the heat equation). Flea migration works a bit like heat transport: they hop around randomly, and after a while they're everywhere. In much the same way as for the heat equation, derive an equation and boundary conditions for the density of fleas $u(\mathbf{x}, t)$ under the following premises:

- The space where fleas hop around is a two-dimensional area ("the room").
- Fleas hop around randomly; consequently if the density of fleas to the right of a line is n times higher than to the left, n times as many fleas will cross it from the right to the left than the other way around (think of what such reasoning meant for the heat flux in relation to the temperature).
- The room has walls through which fleas neither leave not enter.

In the process of deriving your equations, identify the coefficients analogous to heat conductivity, density, etc in the equation you get. State "physical" units for each quantity that appears in your equation and make sure that the equation is dimensionally correct.

Intuitively, since fleas neither leave nor enter the room, their total number must be constant. Can you derive this from the equations? **(6 points)**

Problem 5 (Divergence theorem). For the simple case of the unit square $\Omega = [0, 1]^2$, show that the divergence theorem

$$\int_\Omega \text{div } \mathbf{u} \, dx \, dy = \int_{\partial\Omega} \mathbf{n} \cdot \mathbf{u} \, dl$$

holds for all sufficiently smooth vector fields $\mathbf{u}$. Hint: Use that the integral over Ω is really an integral over $0 \leq x, y \leq 1$ and that in the surface integral on the right you can express the normal vector explicitly on each part of the boundary. **(3 points)**

Problem 6 (Divergence theorem). Solve exercise 1.5.8 in the book. **(2 points)**

Problem 7 (Isocontours of solutions). Solve exercise 1.5.14 in the book. **(2 points)**