

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

$$\begin{array}{ccccccc}
 & \vdots & & \vdots & & \vdots & \\
 & \downarrow & & \downarrow & & \downarrow & \\
 0 & \longrightarrow & A_{n+1} & \xrightarrow{\alpha_{n+1}} & B_{n+1} & \xrightarrow{\beta_{n+1}} & C_{n+1} \longrightarrow 0 \\
 & & \downarrow \partial_{n+1} & & \downarrow \partial'_{n+1} & & \downarrow \partial''_{n+1} \\
 0 & \longrightarrow & A_n & \xrightarrow{\alpha_n} & B_n & \xrightarrow{\beta_n} & C_n \longrightarrow 0 \\
 & & \downarrow \partial_n & & \downarrow \partial'_n & & \downarrow \partial''_n \\
 0 & \longrightarrow & A_{n-1} & \xrightarrow{\alpha_{n-1}} & B_{n-1} & \xrightarrow{\beta_{n-1}} & C_{n-1} \longrightarrow 0 \\
 & & \downarrow \vdots & & \downarrow \vdots & & \downarrow \vdots
 \end{array}$$

$$\begin{array}{ccccc}
 & & & \vdots & \\
 & & \swarrow & & \\
 H_{n+1}(\mathcal{A}) & \xrightarrow{\alpha_*} & H_{n+1}(\mathcal{B}) & \xrightarrow{\beta_*} & H_{n+1}(\mathcal{C}) \\
 & & \swarrow \delta_{n+1} & & \\
 H_n(\mathcal{A}) & \xrightarrow{\alpha_*} & H_n(\mathcal{B}) & \xrightarrow{\beta_*} & H_n(\mathcal{C}) \\
 & & \swarrow \delta_n & & \\
 H_{n-1}(\mathcal{A}) & \xrightarrow{\alpha_*} & H_{n-1}(\mathcal{B}) & \xrightarrow{\beta_*} & H_{n-1}(\mathcal{C}) \\
 & & \swarrow & & \\
 & & & \vdots &
 \end{array}$$

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Homology ($\mathbb{Z}/2\mathbb{Z}$)

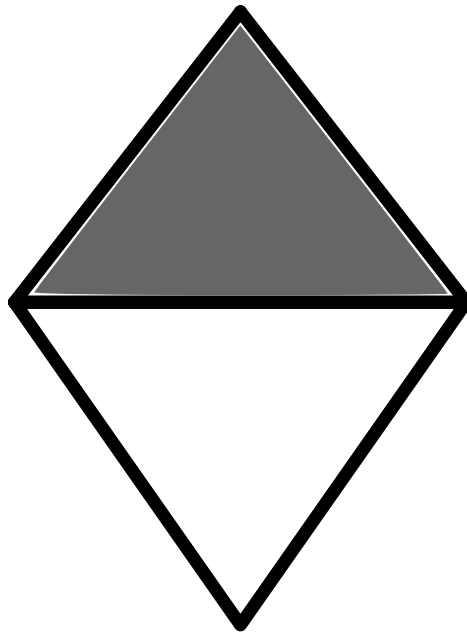
Homology groups $H_0, H_1, H_2, H_3, \dots$

H_k “counts the number of k -dimensional holes”.

Homotopy equivalent shapes have the same homology groups.

Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)



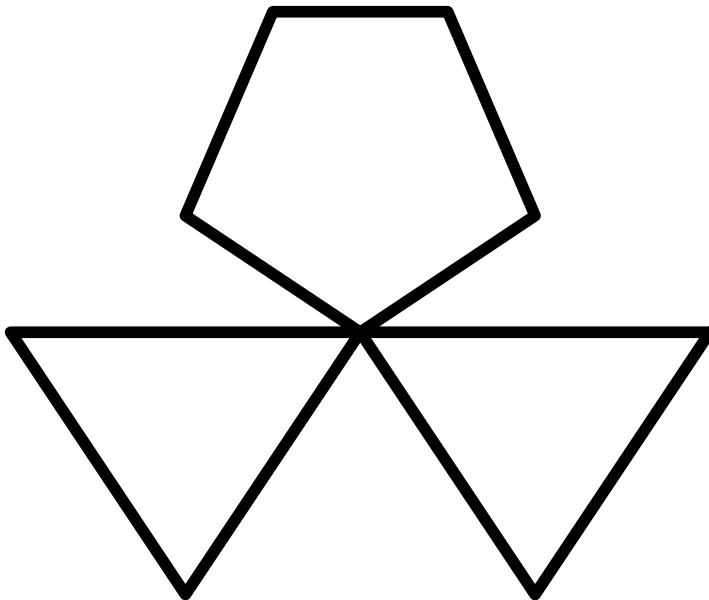
H_0 has rank 1.

H_1 has rank 1.

H_2 has rank 0.

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Homology ($\mathbb{Z}/2\mathbb{Z}$)



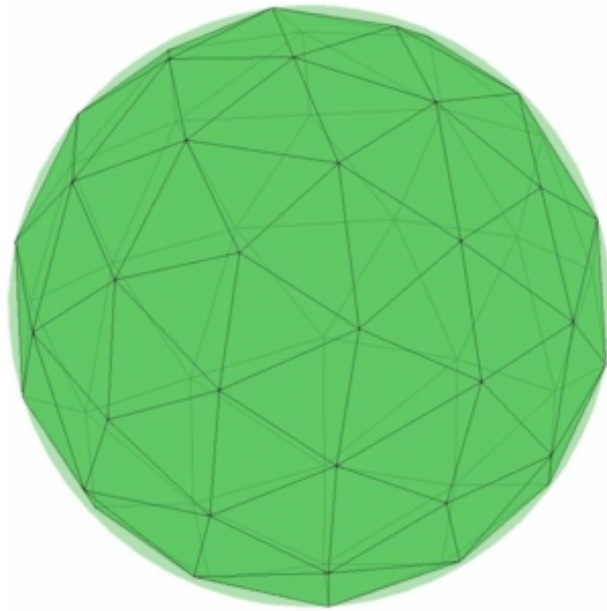
H_0 has rank 1.

H_1 has rank 3.

H_2 has rank 0.

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Homology ($\mathbb{Z}/2\mathbb{Z}$)



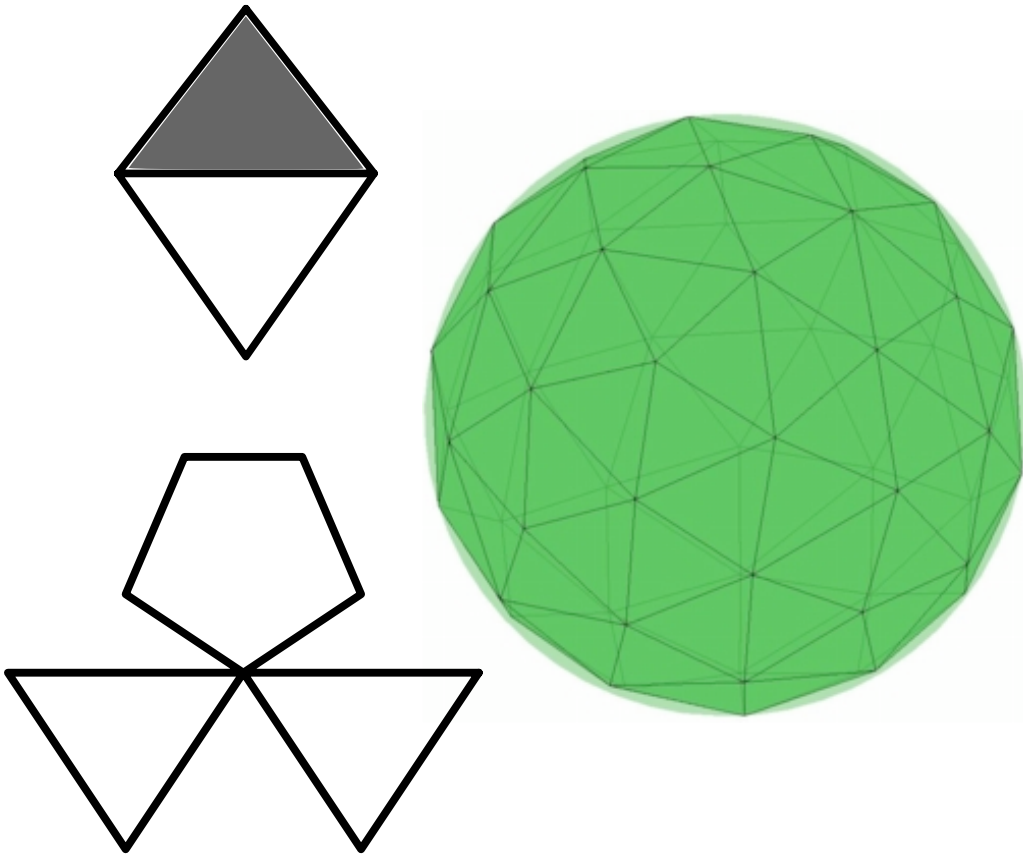
H_0 has rank 1.

H_1 has rank 0.

H_2 has rank 1.

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Homology ($\mathbb{Z}/2\mathbb{Z}$)



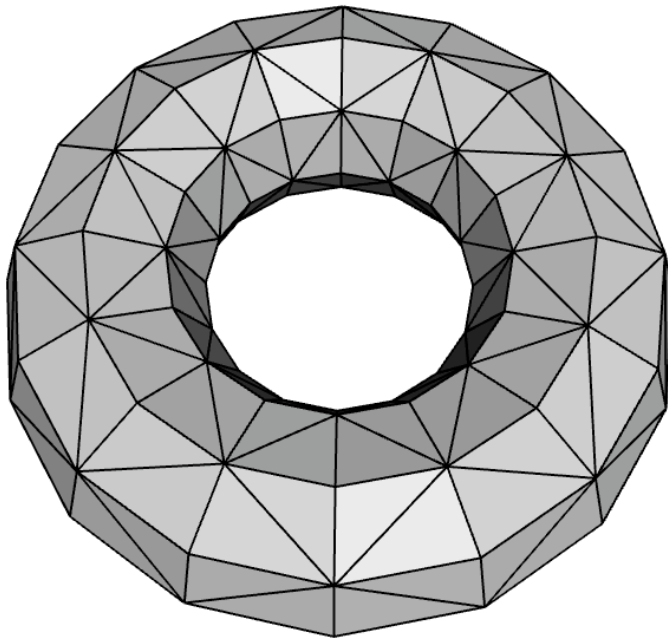
H_0 has rank 3.

H_1 has rank 4.

H_2 has rank 1.

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Homology ($\mathbb{Z}/2\mathbb{Z}$)



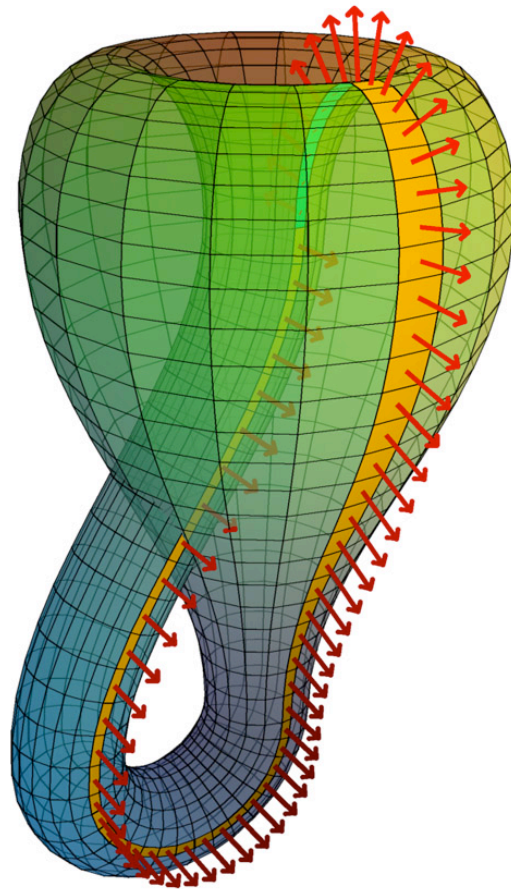
H_0 has rank 1.

H_1 has rank 2.

H_2 has rank 1.

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Homology ($\mathbb{Z}/2\mathbb{Z}$)



H_0 has rank 1.

H_1 has rank 2.

H_2 has rank 1.

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Homology ($\mathbb{Z}/2\mathbb{Z}$)

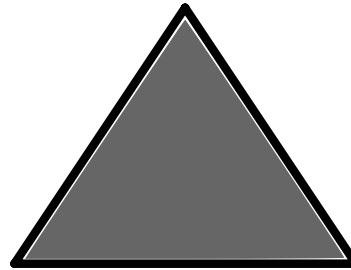
0-simplex



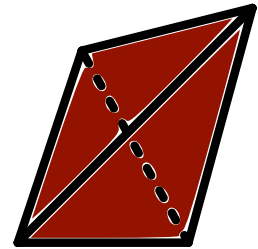
1-simplex



2-simplex



3-simplex



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Homology ($\mathbb{Z}/2\mathbb{Z}$)

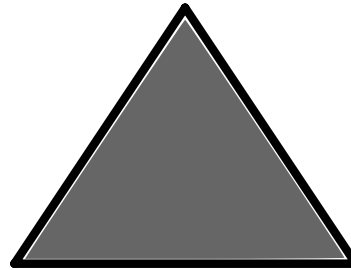
0-simplex



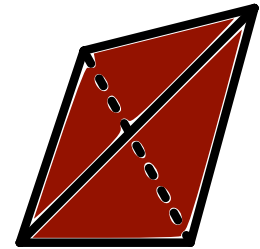
1-simplex



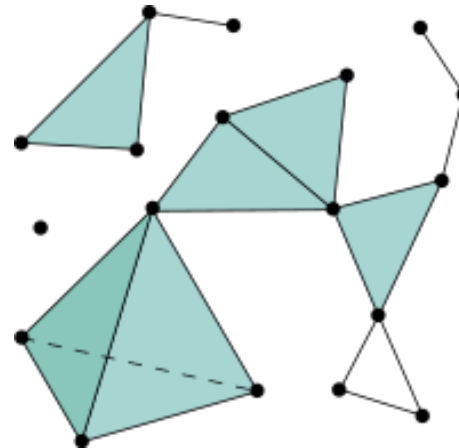
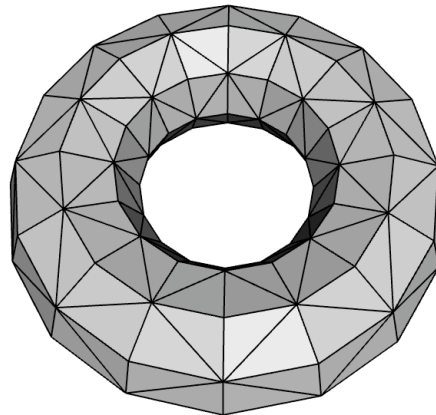
2-simplex



3-simplex



Simplicial complexes



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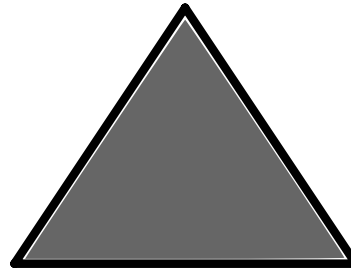
0-simplex



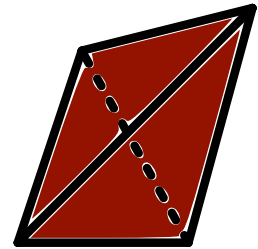
1-simplex



2-simplex



3-simplex



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Homology ($\mathbb{Z}/2\mathbb{Z}$)

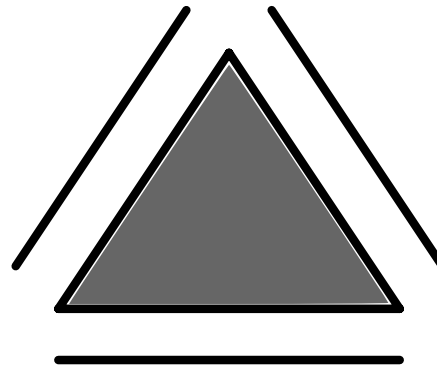
0-simplex



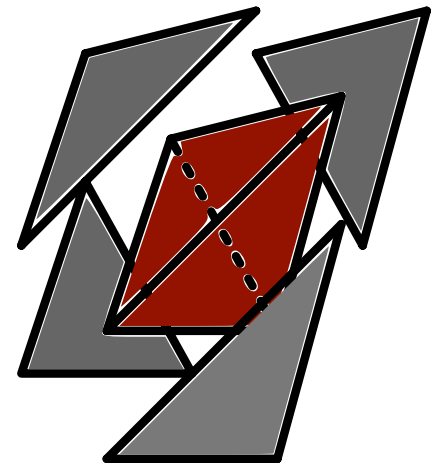
1-simplex



2-simplex



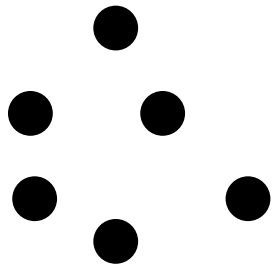
3-simplex



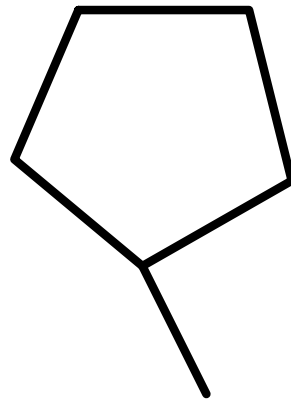
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Homology ($\mathbb{Z}/2\mathbb{Z}$)

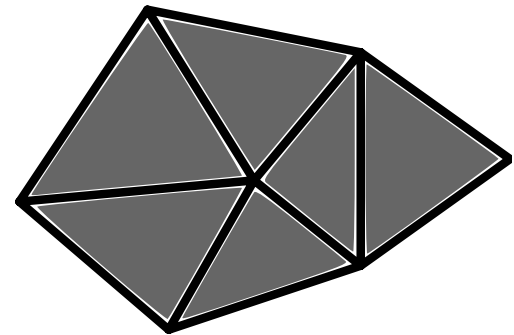
0-simplices



1-simplices



2-simplices



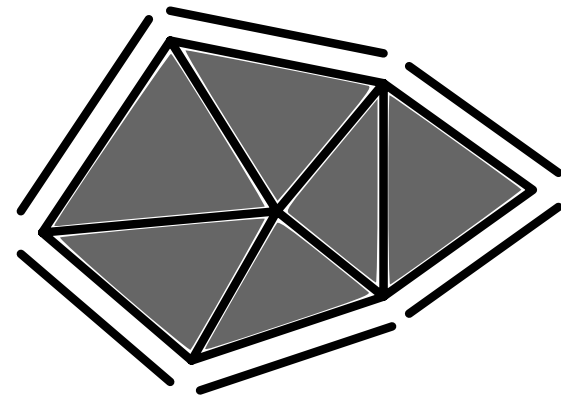
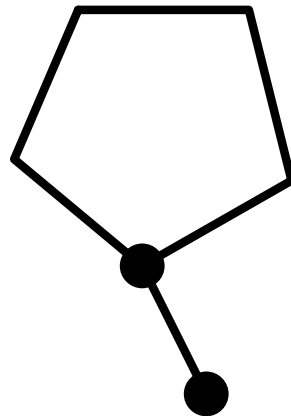
Topology studies shapes

Homology ($\mathbb{Z}/2\mathbb{Z}$)

0-simplices

1-simplices

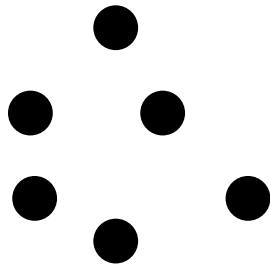
2-simplices



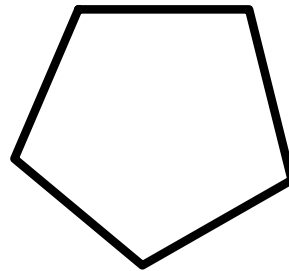
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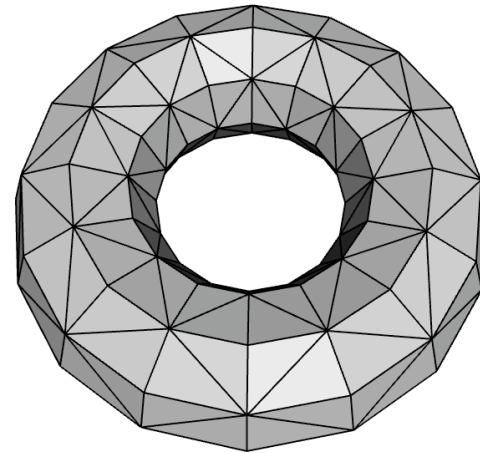
0-cycle



1-cycle



2-cycle

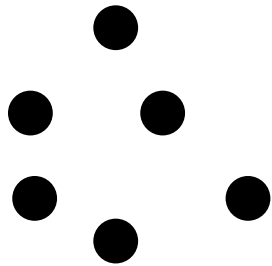


A cycle has no boundary.

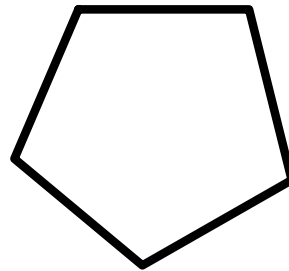
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Homology ($\mathbb{Z}/2\mathbb{Z}$)

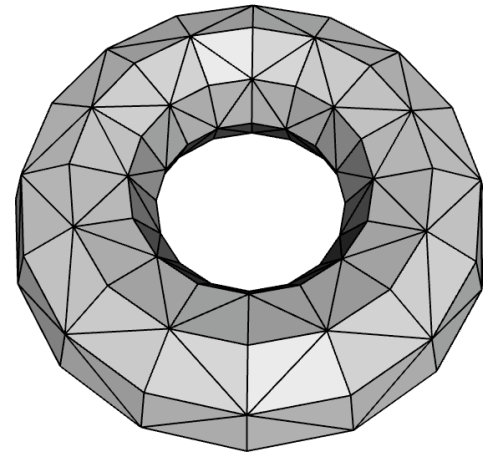
0-cycle



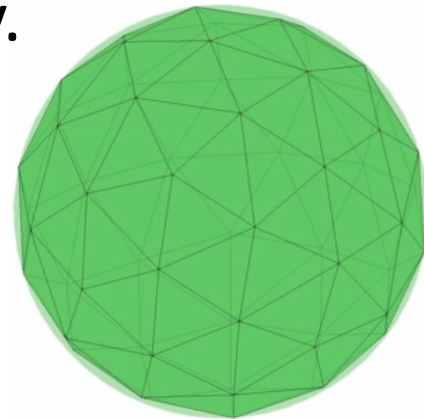
1-cycle



2-cycle

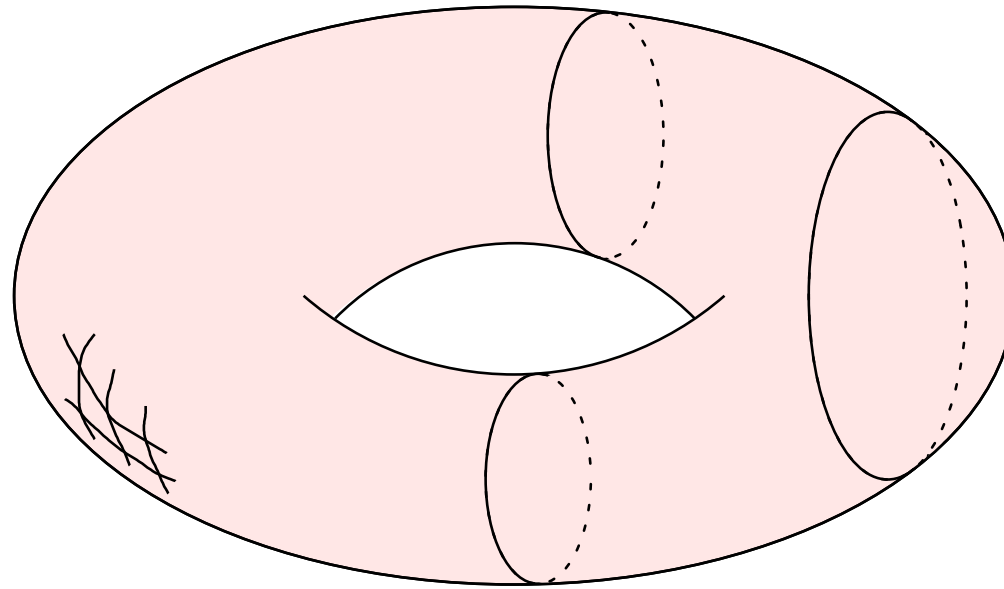


A cycle has no boundary.



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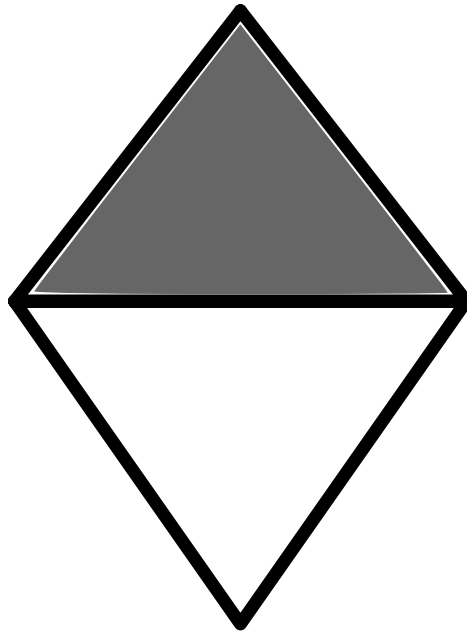


Two cycles are equivalent if they differ by a boundary.

H_k measures equivalence classes of k -cycles.

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H_0 has rank 1.

H_1 has rank 1.

H_2 has rank 0.

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Homology groups $H_0, H_1, H_2, H_3, \dots$

H_k “counts the number of k -dimensional holes”.

Homotopy equivalent shapes have the same homology groups.