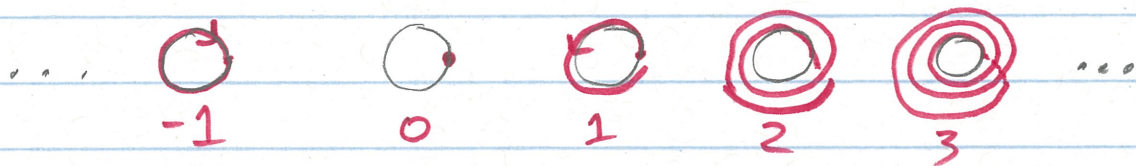


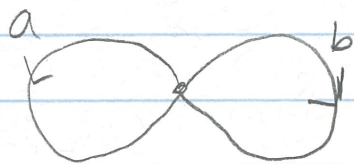
Incorrect Def  $\pi_i(X)$  is the group of homotopy classes of maps  $S^i \rightarrow X$

Ex  $\pi_1(S^1) = \mathbb{Z}$  



Group operation in  $\pi_1$  is loop concatenation

Ex  $\pi_1(\infty) = \pi_1(\square_{a,b}) = \langle a, b \rangle$  is the free group on two generators (non-abelian)



Elements  $ab^2a^{-1}b^{-3}$

Note  $ab^3a^{-2}a^2b = ab^3b = ab^4$

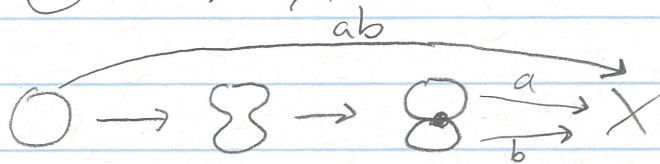
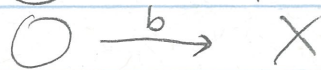
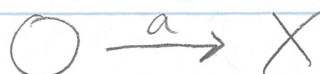
Multiplication  $(aba^2) * (a^{-2}b^{-1}a^2)$

$$= aba^2a^{-2}b^{-1}a^2 = abb^{-1}a^2 = a^3$$

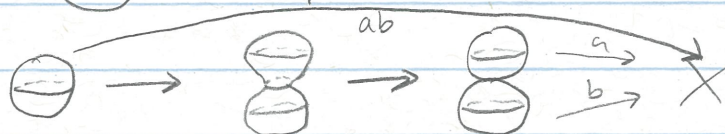
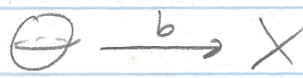
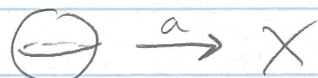
Note  $ab \neq ba$

Ex What's group operation in  $\pi_i$ ?

$\pi_1$



$\pi_2$



Why is  $\pi_i$  abelian for  $i \geq 2$ ?

Data  $\textcircled{a} \xrightarrow{a} X$  is same as  $\begin{matrix} * & & * \\ & a & \\ * & & * \end{matrix} \rightarrow X$

Data  $\textcircled{ab} \xrightarrow{ab} X$  is same as  $\begin{matrix} * & & * \\ & a & \\ * & & * \\ & b & \\ * & & * \end{matrix} \rightarrow X$

Note  $\begin{matrix} a \\ b \end{matrix} \approx \begin{matrix} * & & * \\ & a & \\ * & & * \\ & b & \\ * & & * \end{matrix} \approx \begin{matrix} a \\ * & & b \end{matrix}$

$\approx \begin{matrix} * & & b \\ & a & \\ * & & * \end{matrix} \approx \begin{matrix} b \\ * & & a \end{matrix} \approx \begin{matrix} b \\ a \end{matrix}$