

CSU Math 510

Midterm

Due Friday, October 16 at 8pm

Instructions. Please submit this optional midterm by Friday, Oct 16 at 8pm by emailing it to henry.adams@colostate.edu with the email subject line “Math 510: Midterm”.

Each problem is worth 5 points, and so the exam is out of 20 points.

This an optional midterm, in the sense that it can only improve your grade if you decide to take it.

This midterm was written to be taken in 75 minutes with no books, notes, or calculators (so that you can use it as qualifying exam practice if you want to). But I am not proctoring the exam, and the point of my grading is only to give you feedback on how I will grade the qual. So please feel free to use extra time or outside resources if it will help you learn the material better (in preparation for the qualifying exam).

1. Put the following linear program into equational form (maximize $c^T x$ subject to $Ax = b$ and $x \geq 0$). Note that x_3 is not currently constrained to be nonnegative.

Minimize x_3

subject to

$$\begin{array}{rcccccl} -2x_1 & +x_2 & +2x_3 & & \leq & 1 \\ x_1 & & & -x_3 & +2x_4 & = & 2 \\ x_1 & +x_2 & & & +x_4 & \geq & 3 \\ & & & & & & x_1, x_2, x_4 \geq 0 \end{array}$$

2. Use the simplex method to find an optimal solution to the following linear program. You may choose pivot variables however you like.

Maximize $x_1 + x_2 + x_3 + x_4$

subject to

$$\begin{array}{rccccr} -4x_1 & +x_2 & +x_3 & & & = 10 \\ x_1 & & & +x_4 & & = 3 \\ x_1 & +x_2 & & & & \geq -1 \\ & & & x_1, x_2, x_3, x_4 & & \geq 0 \end{array}$$

3. Use the simplex method on an auxiliary problem to find *any* basic feasible solution to the following linear program. This basic feasible solution does not need to be optimal. You may choose pivot variables however you like.

Remark: Don't just find a basic feasible solution by eye — I am asking you to use the simplex method.

Maximize $x_1 + x_2$

subject to

$$\begin{array}{rclcl} -2x_1 & & +3x_3 & = & 12 \\ x_1 & -x_2 & +x_3 & = & 3 \\ & & x_1, x_2, x_3 & \geq & 0 \end{array}$$

4. The dual of the linear program

Maximize $x_1 + x_2$

subject to

$$\begin{array}{rcll} x_1 & & +x_3 & \leq 5 \\ x_1 & -x_2 & +x_3 & \leq 4 \\ x_1 & +x_2 & & \leq 6 \\ & & x_1, x_2, x_3 & \geq 0 \end{array}$$

is:

Minimize $5y_1 + 4y_2 + 6y_3$

subject to

$$\begin{array}{rcll} y_1 & +y_2 & +y_3 & \geq 1 \\ & -y_2 & +y_3 & \geq 1 \\ y_1 & +y_2 & & \geq 0 \\ & & y_1, y_2, y_3 & \geq 0 \end{array}$$

Show how this dual linear program is derived, briefly explaining your logic.

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