CSU Math 510

Homework 1

Due Tuesday, September 22 at 8pm

Instructions. Of the following problems, choose any two (or more) to do. Alternatively, do zero of these problems, and instead do something else that is beneficial to you! That "something else" might be:

- Do a cooler problem of your choosing.
- Relate the class to your research.
- Learn a side-topic related to linear programming or optimization.
- Code something up.
- Make a blog post.
- Make a YouTube video.
- etc.

Please submit this homework by emailing it to `henry.adams@colostate.edu` with the email subject line "Math 510: Homework 1".

I may take one or two of your answers and use them as "solutions" that I share with the class. I will only share your answer in a positive light, not in a negative light. But if you'd rather I not use any of your answers for this purpose, simply let me know!

Problems.

1. Read the 13 page article *Linear Programming: The Story about How it Began* by George Dantzig.

2. Solve a linear programming problem of your choosing in a software package of your choosing (perhaps python, Matlab, etc). There should be built-in commands for doing this.

3. Find and briefly describe a cool example application of linear programming.

4. A set C in $\mathbb{R}^n$ is *convex* if given any two points $x, y \in C$, the entire line-segment connecting x and y is also contained in C. Prove that if C_1 and C_2 are two convex sets in $\mathbb{R}^n$, then their intersection $C_1 \cap C_2$ is also convex.

5. Find the vertices of the polytope in $\mathbb{R}^3$ that is defined by the following system of linear inequalities:

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 4 \\ x_1 &\leq 2 \\ x_3 &\leq 3 \\ 3x_2 + x_3 &\leq 6 \\ x_1 + 2x_2 + 2x_3 &\leq 9 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \\ x_3 &\geq 0 \end{aligned}$$

   Are any of the above linear inequalities redundant, meaning they can be removed without changing the polytope?

   Remark: You have not been taught an algorithm for doing this. But, c'mon, it feels like we might be able to play around and figure it out, right? Exploring in mathematics builds your intuition and leads to good ideas!

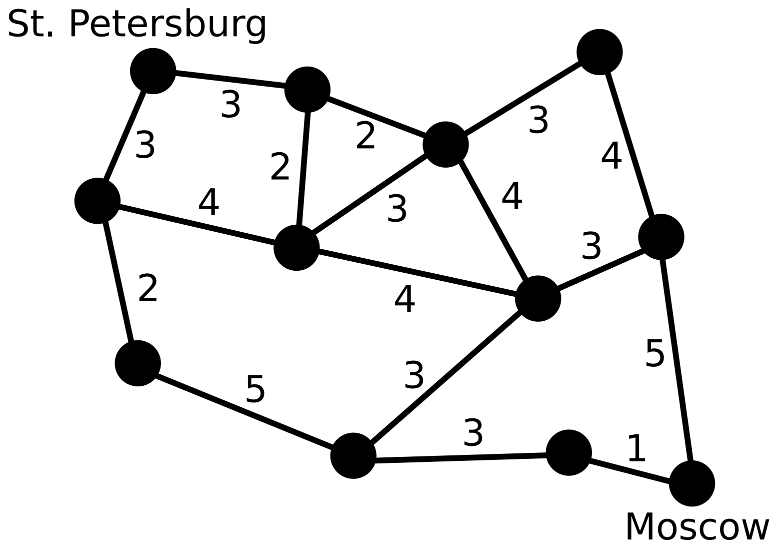
6. Find the half-spaces which define the polytope in $\mathbb{R}^3$ that is defined as the convex hull of the following set of points:

$$\{(0,0,0), (-2,1,-3), (3,-4,2), (1,-2,5), (-1,-3,4), (-2,-2,-2),$$
$$(3,5,2), (1,2,-2), (2,1,-3), (-1,1,3), (-4,2,-1), (2,-2,-1)\}.$$

   Are any of the points above redundant, meaning they can be removed without changing the polytope?

   Remark: You have not been taught an algorithm for doing this. But, c'mon, it feels like we might be able to play around and figure it out, right? Exploring in mathematics builds your intuition and leads to good ideas!

7. Watch a YouTube video on the Ford-Fulkerson algorithm for finding the maximum flow in a network, and then use this algorithm to find the maximum flow from St. Petersburg to Moscow in the below network.



8. Any other problem of your choosing!