CSU Math 510

Midterm

Instructions. Please submit this optional final / qualifying exam by Monday, Dec 14 at 11:40am by emailing it to `henry.adams@colostate.edu`.

There are 6 problems, and each problem is worth 10 points. So the exam is out of 60 points.

This exam is to be taken in 120 minutes with no books, notes, or calculators.

Please sign below to indicate you accept the following statement:

"I will not give, receive, or use any unauthorized assistance, and in particular I will not communicate or collaborate with anybody during the exam."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1. Use the simplex method to find an optimal solution to the following linear program. If the problem is unfeasible or if the optimization function is unbounded from above, then use the simplex method to deduce and show this.

   Maximize $x_1 + 2x_2 - x_3$
   subject to

$$\begin{array}{llll} -4x_1 & +x_2 & +x_3 & = 10 \\ x_1 & & +x_3 & = 3 \\ & & x_1, x_2, x_3 & \geq 0 \end{array}$$

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2. Write down the dual of the following linear program. After stating what the dual linear program is, show how the dual linear program is derived, explaining your logic.

   Maximize $x_1 + 3x_3$
   subject to

$$\begin{array}{llll} x_1 & +x_2 & +x_3 & \leq 2 \\ x_1 & -2x_2 & +x_3 & \leq -1 \\ x_1 & +3x_2 & & \leq 4 \\ & & x_1, x_2, x_3 & \geq 0 \end{array}$$

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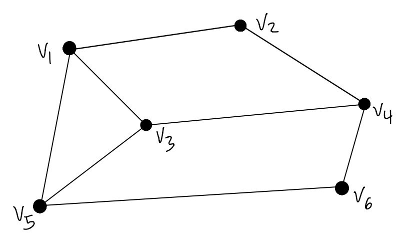
3. A *maximum independent set* in a graph is a collection of vertices in the graph, no two of which are adjacent, such that the number of vertices is as large as possible.

   (a) Write down, but do not solve, an integer linear programming problem whose solution gives a maximum independent set for the following graph. Briefly explain the notation for the variables you use.



(b) The *linear relaxation* of the above problem is obtained by dropping the integrality constraint. Explain why for general graphs, the linear relaxation may give an arbitrarily bad approximation for the size of a maximum independent set. One way to do this is by describing a family of graphs where the linear relaxation approximation can be arbitrarily bad.

4. Argue why finding an optimal solution to a linear programming problem is in some sense no harder than finding any feasible solution. In this class we mentioned three such possible arguments, one of which involved a binary search, and a second of which involved the duality of linear programming.

5. Explain why under many different choices of pivot rule, it is plausible that the simplex algorithm has exponential running time in the worst case. (This is known to be true for some choices of pivot rule, and suspected to be true for most other pivot rules.) I am not asking for a proof, but instead a plausibility argument.

6. Prove that if a linear programming problem admits an optimal solution, then it also admits an optimal solution at one of the vertices of the polytope defining the feasible region.

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