CSU Math 472
Fall 2022

Homework C

Due Wednesday, December 7, in class (else by email)

Problems.

1. Let $D \subseteq \mathbb{R}^2$ be the unit disc bounded by S^1, parametrize D using polar coordinates, and let $f\colon D \to D$ be the continuous function defined by $f(r, \theta) = (r, \theta + 2\pi r)$. Find a homotopy $H\colon D \times I \to D$ from f to the identity map on D.

2. (a) Take four edges which form a square (but not the interior of the square), and identify the left and right edges and the top and bottom edges, as you would do to form a torus (if the interior of the square was included). Explain what the resulting space is, and explain why the fundamental group is $\langle a, b\rangle$, the free group on two generators.

   (b) Take a square, and identify the left and right edges and the top and bottom edges to form a torus. Explain why the fundamental group of the torus is $\langle a, b \mid aba^{-1}b^{-1}\rangle$, which is isomorphic to $\mathbb{Z} \times \mathbb{Z}$.

3. Explain why the fundamental group of the Klein bottle is isomorphic to $\langle a, b \mid aba^{-1}b\rangle$ (or, if you prefer, to $\langle a, b \mid abab^{-1}\rangle$ — these two groups are isomorphic).

4. Let S^1 be the unit circle and let $C = S^1 \times [-1, 1]$ be a cylinder. Prove that $S^1 \simeq C$.