CSU Math 472
Fall 2022

Homework A

Due Wednesday, October 5, in class (else by email)

Problems.

1. Define $f\colon [0, 2\pi) \to S^1 = \{(x, y) \mid x^2 + y^2 = 1\}$ by $f(t) = e^{it} = (\cos(t), \sin(t))$. The function f is bijective and continuous. Show that the function $f^{-1}\colon S^1 \to [0, 2\pi)$ is not continuous. (This means that f is not a homeomorphism.)

2. (a) Prove that a finite union of closed sets is closed.

   (What follows is an equivalent statement of this problem that provides useful notation. Let X be a topological space, and let $C_1, C_2, \ldots, C_n$ be a finite collection of closed sets in X. Prove that $\cup_{i=1}^{n} C_i$ is closed in X.)

   (b) Find a (necessarily infinite) collection of closed sets in some topological space whose union is not closed.

3. For each of the following sets $A \subseteq \mathbb{R}^2$, state (without proof) what the closure $\overline{A}$ is, state (without proof) what $\overline{\mathbb{R}^2 \setminus A}$ is, and from there derive what ∂A is.

   (a) $A = \{(x, y) \in \mathbb{R}^2 \mid x \neq 0 \text{ and } y \neq 0\}$.

   (b) $A = \{(x, y) \in \mathbb{R}^2 \mid -1 \leq x \leq 1 \text{ and } -2 < y < 2\}$.

   (c) $A = \{(t, \sin(1/t)) \mid t > 0\}$.

4. The cover of our textbook shows a hollow two-holed torus whose arms are “linked.” Show that the arms are in fact not really linked, by drawing a sequence of pictures illustrating a deformation that stretches and bends (but does not tear or break) the two-holed torus into a configuration where the arms are not linked.

5. Let X, Y, and Z be topological spaces. Use the definition of a homeomorphism to show that if X is homeomorphic to Y and Y is homeomorphic to Z, then X is homeomorphic to Z.