Duke Math 431
Spring 2015

Proof of Corollary 3.3.6

Corollary 3.3.6. Let f be a continuous function on the interval $[a, b]$. Then,

$$\left| \int_a^b f(x)dx \right| \leq (b-a) \sup_{[a,b]} |f(x)|.$$

Proof. We have

$$\left| \int_a^b f(x)dx \right| \leq \int_a^b |f(x)|dx \qquad \text{by Theorem 3.3.5}$$

$$\leq \int_a^b \sup_{[a,b]} |f(x)|dx \qquad \text{by Theorem 3.3.4, since } f(x) \leq \sup_{[a,b]} |f(x)|$$

$$= \sup_{[a,b]} |f(x)| \int_a^b 1dx \qquad \text{by Theorem 3.3.3, since } \sup_{[a,b]} |f(x)| \text{ is a constant}$$

$$= (b-a) \sup_{[a,b]} |f(x)|.$$

$\square$