Duke Math 431 **Practice Midterm 2** **March 21, 2015**

Name: ____

- This is the Practice Midterm 2 for Duke Math 431. Partial credit is available. No notes, books, calculators, or other electronic devices are permitted.
- Write proofs that consist of complete sentences, make your logic clear, and justify all conclusions that you make.
- Please sign below to indicate you accept the following statement:

  "I have abided with all aspects of the honor code on this examination."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1 (a) Give an example of a function f and a domain D so that $f\colon D \to \mathbb{R}$ is continuous but not uniformly continuous. No proofs are necessary.

(b) Give an example of a function $f\colon [0,1] \to \mathbb{R}$ that is not Riemann integrable.

(c) Suppose f is n times continuously differentiable on $[a,b]$ and that $f^{(n+1)}$ exists. Let $T^{(n)}(x, x_0)$ be the n-th Taylor polynomial of f at x_0. State the conclusion of Taylor's Theorem.

2 Suppose $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ are continuously differentiable, that $f(0) = g(0)$, and that $f'(x) \le g'(x)$ for all $x \ge 0$. Prove that $f(x) \le g(x)$ for all $x \ge 0$.

3 Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is continuously differentiable and that its derivative f' is bounded. Prove that f is uniformly continuous on $\mathbb{R}$.

4 Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable at x. Prove that $\lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h} = f'(x)$.

5 Let $f\colon [0,3] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & 1 < x < 2 \\ 2 & x = 2 \\ 1 & 2 < x \le 3 \end{cases}$$

Prove that f is Rieman integrable and compute $\int_0^3 f(x)dx$.

6 Let f be a continuous function on the interval $[a, b]$. Suppose that for every $c \in [a, b]$ and $d \in [a, b]$ we know that $\int_c^d f(x)dx = 0$. Prove that $f(x) = 0$ for all x.