Duke Math 431 **Practice Midterm 1** **February 9, 2015**

Name: ______________________________

- This is the Practice Midterm 1 for Duke Math 431. Partial credit is available. No notes, books, calculators, or other electronic devices are permitted.
- Write proofs that consist of complete sentences, make your logic clear, and justify all conclusions that you make.
- Please sign below to indicate you accept the following statement:

  "I have abided with all aspects of the honor code on this examination."

Signature: ______________________________

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

1 (a) Let $\{a_n\}$ be a sequence of real numbers. Give the precise definition of when $\{a_n\}$ converges to some limit $a \in \mathbb{R}$.

(b) Let $\{a_n\}$ be a sequence of real numbers. Give the precise definition of when $\{a_n\}$ has $d \in \mathbb{R}$ as a limit point.

2 Show that the sequence $\{a_n\}$ given by $a_n = 5 + \frac{2}{\sqrt[3]{n}}$ satisfies the definition of a Cauchy sequence.

3 Let A, B, and C be sets. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

4 Let $\{a_n\}$ be a sequence of real numbers and let S be a set of real numbers. Suppose that a_n is an upper bound for S for each $n \in \mathbb{N}$, and that $a_n \to a$. Prove that a is an upper bound for S.

5 Suppose that $a_n \to 0$ and $\{b_n\}$ is bounded. Prove that $a_n b_n \to 0$.

6 For the following true and false questions, you do not need to explain your answer at all. Just write "True" or "False".

(a) True or false: There exists a one-to-one function $f\colon \mathbb{Q} \times \mathbb{Q} \to \mathbb{N}$.

(b) True or false: If a sequence $\{a_n\}$ is not bounded, then it either diverges to $+\infty$ or diverges to $-\infty$.

(c) True or false: If r_1 and r_2 are irrational numbers with $r_1 < r_2$, then there exists a rational number q satisfying $r_1 < q < r_2$.

(d) True or false: If a sequence $\{a_n\}$ has exactly one limit point d, then sequence $\{a_n\}$ converges to d.

(e) True or false: If function $f\colon S \to T$ is one-to-one, then its inverse function $f^{-1}\colon \mathrm{Ran}(f) \to S$ is one-to-one.