

Name: _____

- This is Midterm 2 for Duke Math 431. Partial credit is available. No notes, books, calculators, or other electronic devices are permitted.
- Write proofs that consist of complete sentences, make your logic clear, and justify all conclusions that you make.
- Please sign below to indicate you accept the following statement:
“I have abided with all aspects of the honor code on this examination.”

Signature: _____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

- 1 (a) Give the precise definition of when a function $f: Dom(f) \rightarrow \mathbb{R}$ is uniformly continuous. (4 points)

- (b) Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is not uniformly continuous on \mathbb{R} . (6 points)

- 2 (a) Compute the fourth-order Taylor polynomial $T^{(4)}(x, 0)$ for $f(x) = \cos x$ about $x = 0$. (4 points)

- (b) Use Taylor's Theorem to prove $\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$ exists. (6 points)

- 3 Suppose $f: [0, 1) \rightarrow \mathbb{R}$ is a continuously differentiable function that is not bounded, and for simplicity assume $f(0) = 0$. Prove that $f': [0, 1) \rightarrow \mathbb{R}$ is not bounded.

4 Define $f: [0, 3] \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x \leq 2 \\ 2 & \text{if } 2 < x \leq 3. \end{cases}$$

Prove that f is Riemann integrable.

5 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ via

$$f(x) = \begin{cases} x^2 \cos(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

- (a) For $x \neq 0$, compute $f'(x)$.
- (b) Compute $f'(0)$.
- (c) Prove f' is not continuous at $x = 0$.

- 6 State Rolle's Theorem (along with its hypotheses) for a function $g: [a, b] \rightarrow \mathbb{R}$.
State the Mean Value Theorem (along with its hypotheses) for a function $f: [a, b] \rightarrow \mathbb{R}$.
Use Rolle's Theorem to prove the Mean Value Theorem.