Duke Math 431
Spring 2015

Homework 3

Due Friday, January 30

Reading. Sections 2.1, 2.2, and 2.4.

Problems.
Section 1.4: #8, 9.

Section 2.1: #2(b,d), 3(b,d), 5, 9.

Additional Problem: The Submarine Riddle (Part II).
Consider the following math riddle, in which our goal is to destroy an enemy submarine. At each time $t \in \mathbb{N} = \{1, 2, 3, \ldots\}$ the submarine is located at an integer on the number line, and moves as follows. The submarine has an integer initial position $p \in \mathbb{Z}$ and a constant integer velocity $v \in \mathbb{Z}$. So at time 1 the submarine's position is $p + v$, at time 2 its position is $p + 2v$, $\ldots$, and at time t its position is $p + tv$. The values of the integers p and v are unknown to us. At each time $t \in \mathbb{N}$ we get to fire a missile at a single integer position on the number line. Can we devise a strategy for firing our missiles so that we are guaranteed to eventually hit the enemy submarine with unknown initial position p and velocity v? That is, can we devise a strategy for firing our missiles so that we eventually hit every possible enemy submarine? Note that a firing strategy is a function $f \colon \mathbb{N} \to \mathbb{Z}$, where $f(t) \in \mathbb{Z}$ is the location where we fire our missile at time $t \in \mathbb{N}$.

(a) Recall that if S and T are countable sets, then $S \times T$ is countable (this is Proposition 1.3.4 in the book). Hence there exists a one-to-one and onto function $h \colon \mathbb{N} \to \mathbb{Z} \times \mathbb{Z}$. Draw pictures of two such possible functions h (recall the spiraling pictures from lecture), and label $h(1)$, $h(2)$, $\ldots$, $h(10)$ in these pictures.
(b) As a corollary, show there exists a firing strategy $f \colon \mathbb{N} \to \mathbb{Z}$ which eventually hits any possible submarine with initial position $p \in \mathbb{Z}$ and constant velocity $v \in \mathbb{Z}$.