Duke Math 431
Spring 2015

Homework 2

Due Friday, January 23

Reading. Sections 1.4 and 2.1.

Problems.
Section 1.3: #1, 3(c).
For 3(c) you may assume the two sets are disjoint, i.e. they have no elements in common.

Section 1.4: #10, 11(c).
For 10(c), let $f^2(x) = f(x)f(x)$.
For 11(c), I suggest finding a rational number q *with* $c - \sqrt{2} < q < d - \sqrt{2}$ *and then using Extra Problem 1 below.*

Extra Problem 1.
(a) For q and r rational, prove $q + r$ is rational.
(b) For q rational and r irrational, prove $q + r$ is irrational.
(c) Give an example where q and r are irrational and $q + r$ is rational.

Extra Problem 2: The Submarine Riddle (Part I).
Consider the following math riddle, in which our goal is to destroy an enemy submarine. At each time $t \in \mathbb{N} = \{1, 2, 3, \ldots\}$ the submarine is located at an integer on the number line, and moves as follows. The submarine has an integer initial position $p \in \mathbb{Z}$ and a constant integer velocity $v \in \mathbb{Z}$. So at time 1 the submarine's position is $p + v$, at time 2 its position is $p + 2v$, ..., and at time t its position is $p + tv$. The values of the integers p and v are unknown to us. At each time $t \in \mathbb{N}$ we get to fire a missile at a single integer position on the number line. Can we devise a strategy for firing our missiles so that we are guaranteed to eventually hit the enemy submarine with unknown initial position p and velocity v? That is, can we devise a strategy for firing our missiles so that we eventually hit every possible enemy submarine? Note that a firing strategy is a function $f \colon \mathbb{N} \to \mathbb{Z}$, where $f(t) \in \mathbb{Z}$ is the location where we fire our missile at time $t \in \mathbb{N}$.

On Homework 3 we will solve this riddle completely, but for now we consider only two simplified cases.
(a) Prove $\mathbb{Z}$ is countable by giving a one-to-one and onto function $h \colon \mathbb{N} \to \mathbb{Z}$.
(b) Show how to solve the submarine riddle if the initial position $p \in \mathbb{Z}$ is arbitrary but we add the simplifying assumption that the velocity v is zero.
(c) Show how to solve the submarine riddle if the velocity $v \in \mathbb{Z}$ is arbitrary but we add the simplifying assumption that the initial position p is zero.
Hint: For (b) and (c), use function $h \colon \mathbb{N} \to \mathbb{Z}$ *from (a) to give a firing strategy* $f \colon \mathbb{N} \to \mathbb{Z}$.