CSU Math 366

Homework 6

Due Friday, April 30, anytime, on Gradescope.

Remark. Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words.

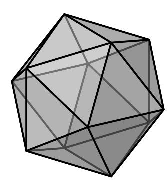
Reading. Read Chapters 16 and 17.

Problems.

1. What is the complete list of abelian groups of size $1008 = 2^4 \cdot 3^2 \cdot 7$ up to isomorphism?

   Remark: Any abelian group of 1008 should be isomorphic to exactly one of the groups on your list.

2. An icosahedron (pictured below) is a Platonic solid with 20 triangular faces and 12 vertices. Let G be the group of rotational symmetries of an icosahedron.

   (a) Let S be the set of 20 faces. Think of G as a permutation group acting on S. Fix a face $i \in S$. What is the size of the stabilizer of i? What is the size of the orbit of i? Use the orbit-stabilizer theorem to deduce the size of group G.

   (b) Let S be the set of 12 vertices. Think of G as a permutation group acting on S. Fix a vertex $i \in S$. What is the size of the stabilizer of i? What is the size of the orbit of i? Use the orbit-stabilizer theorem to deduce the size of group G.

3. Let H be a subgroup of group G, and let $a \in G$.

   (i) Give an example to show that it is possible to have $aH \neq Ha$, i.e., the left coset of H in G by a may not be equal to the right coset of H in G by a (write out the details of this example).

   (ii) Prove that if G is abelian, then necessarily $aH = Ha$, i.e. H is *normal* in G.

4. Let H be the subgroup of $G = \mathbb{Z}/8\mathbb{Z}$ given by $H = \langle 4 \rangle = \{0, 4\}$. Say why H is normal in G. Draw the Cayley table for the quotient group G/H.