CSU Math 366 **Practice Midterm 2B**

Name: ________________________________

- For #1, no justification is necessary — the right answer alone suffices. However, extra explanation may help you get more partial credit in the presence of mistakes.

  For #2, #3, and #4, explain your logic fully and write complete sentences.

  For #5, just say "True" or "False". No partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:

  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 (a) (3 points) Write the element $\alpha = (1523)(235)(12) \in S_5$ in disjoint cycle form.

(b) (3 points) Write $\beta = (1248)(73982) \in S_9$ as a product of (not necessarily disjoint) 2-cycles. Is β an element of the alternating group A_9?

(c) (4 points) If $\gamma = (1432)(2453) \in S_5$, then write its inverse γ^{-1} in disjoint cycle form.

2 Let G be a group, and let $g \in G$.

(a) (3 points) Define $\langle g \rangle$, the *cyclic subgroup generated by* g.

(b) (7 points) Prove that $\langle g \rangle$ is a subgroup of G.

3 (a) (3 points) Define what it means for a subgroup H of G to be *normal* in G.

(b) (7 points) Prove that if H is a subroup of G with $|G|/|H| = 2$, then H is normal in G.

4 (a) (3 points) Define what it means for a function $\phi\colon G \to \overline{G}$ between groups G and $\overline{G}$ to be an *isomorphism*.

(b) (7 points) Let $\phi\colon G \to \overline{G}$ be an isomorphism. Prove that if G is abelian, then $\overline{G}$ is also abelian.

[5] No justification needed: just say "True" or "False". No partial credit.

(a) True or False: If H and H' are normal subgroups of a group G, then $|H| = |H'|$.

(b) True or False: There is an isomorphism from the dihedral group D_6 to the alternating group A_4.

(c) True or False: If p is a prime number, then $|\text{Aut}(\mathbb{Z}/p\mathbb{Z})| = |U(p)| = p - 1$.

(d) True or False: Recall S_n is the group of permutations of $\{1, 2, \ldots, n\}$.
For any $i \in \{1, 2, \ldots, n\}$, we have that $n! = n \cdot |\text{stab}_{S_n}(i)|$, which therefore gives $|\text{stab}_{S_n}(i)| = (n-1)!$.

(e) True or False: Any quotient group of an abelian group is also abelian.

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