CSU Math 366 **Practice Midterm 2A**

Name: ____

- For #1, #2, #3, and #4, explain your logic fully and write complete sentences. For #5, just say "True" or "False". No partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement: "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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1 (a) (3 points) State what it means for a function $\phi\colon G \to \overline{G}$ between two groups to be a *homomorphism*.

(b) (4 points) Let $\phi\colon G \to \overline{G}$ be a homomorphism. Prove that $\phi(id_G) = id_{\overline{G}}$.

(c) (3 points) Let $\phi\colon G \to \overline{G}$ be a homomorphism. Prove that for any element $g \in G$, we have that $\phi(g^{-1}) = (\phi(g))^{-1}$.

2 (a) (5 points) Let G be a group and let H be a normal subgroup of G. Define the quotient group G/H (say what the elements of G/H are, and say how the binary operation on G/H is defined).

(b) (5 points Let $G = \mathbb{Z}/15\mathbb{Z}$, and let $H = \langle 3 \rangle = \{0, 3, 6, 9, 12\}$. You may take it as a fact that H is normal in G. List the distinct left cosets of H in G (along with the elements in each such coset), and draw the Cayley table for G/H.

3 Let G be a group of permutations of a set S.

(a) Define the *stabilizer* $\text{stab}_G(i)$ of an element $i \in S$.
Hint: This should be a subset of G (it turns out to also be a subgroup of G)

(b) Define the *orbit* $\text{orb}_G(i)$ of an element $i \in S$.
Hint: This should be a subset of S.

(c) Finish the following theorem: Let G be a finite group of permutations of a set S. The Orbit-Stabilizer Theorem states that for any $i \in S$, we have ....

(d) Use the Orbit-Stabilizer Theorem to deduce the size of the group G of rotational symmetries of a soccer ball (a soccer ball has 20 hexagons and 12 pentagons).

4 Lagrange's Theorem states that if G is a finite group and H is a subgroup, then $|H|$ divides $|G|$, and furthermore, $|G|/|H|$ is equal to the number of distinct left cosets of H in G. Using standard properties of cosets (state these properties as you use them), give a proof of Lagrange's Theorem.

5 No justification needed: just say "True" or "False". No partial credit.

(a) True or False: If G is a finite group, if H and K are subgroups of G, and if $x, y \in G$, then the cosets xH and yK necessarily have the same size.

(b) True or False: $\{id, (23)\}$ is a normal subgroup of S_3.

(c) True or False: Every automorphism is a homomorphism.

(d) True or False: If $\phi\colon G \to \overline{G}$ is a homomorphism between finite groups, and if H is a subgroup of G, then $\phi(H)$ is a subgroup of $\overline{G}$ with $|H| = |\phi(H)|$.

(e) True or False: If $\phi\colon G \to H$ is a homomorphism and $|G| = |H|$, then ϕ is an isomorphism.

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