**CSU Math 366** **Practice Midterm 1B**

Name: _____

- For #1, #2, #3, and #4, explain your logic fully and write complete sentences. For #5, just say "True" or "False". No partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement: "I will not give, receive, or use any unauthorized assistance."

Signature: _____

| Problem | Total Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 50 | |

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1 (a) (6 points) Define what it means for a set $G$ to be a group.

*Remark: If you use a word such as "identity", for example, then you should briefly say what an identity is.*

(b) (4 points) Give an example of a group that is not commutative. Briefly say what the elements in that group are, briefly say what the operation in that group is, and pick two elements in that group and show that those elements do not commute.

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2 (a) (3 points) For $G$ a group and for $a \in G$, define what it means for $b \in G$ to be an *inverse* of $a$.

(b) (7 points) Prove that if $G$ is a group and $a$ is an element in $G$, then $a$ has a unique inverse in $G$.

3 Write down the complete list of subgroups of $\mathbb{Z}_{15}$, sorted from the subgroup with the smallest order to the subgroup with the largest order. Each subgroup should appear once and only once. List the order of each subgroup, and for each subgroup give a single element that generates that subgroup.

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4 Prove that if $\gcd(k, n) = 1$, then $k \in \mathbb{Z}_n$ generates $\mathbb{Z}_n$.

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5 No justification needed: just say "True" or "False". No partial credit.

(a) True or False: If $G$ is a group with $a, b \in G$, then $(ab)^{-1} = a^{-1}b^{-1}$.

(b) True or False: $\mathbb{Z}_{10}$ is a subgroup of $\mathbb{Z}$.

(c) True or False: The group $U(n)$ is a cyclic group generated by any element $k \in U(n)$ that is relatively prime to $n$.
*Recall $U(n)$ consists of the numbers from 1 to $n-1$ that are relatively prime to $n$, with group operation multiplication mod $n$.*

(d) True or False: In the symmetric group $S_4$, we have $(13)(142)(24) = (24)(142)(13)$.

(e) True or False: In the symmetric group $S_4$, the order of the element $(132)(243)(12)(312)$ is 4.

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