CSU Math 366 **Midterm 2**

Name: ____

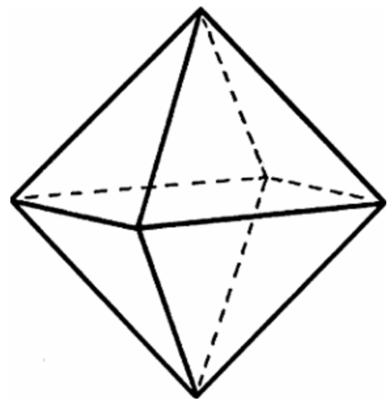
- For all problems, fully explain your answers.
- You do not need to print out this exam; write on whatever paper you like with the problem numbers clearly labeled. **Submit photos of your completed exam on Gradescope.** If (and only if) you have technical difficulties with gradescope, please email me your exam. You have 90 minutes to do the exam; for most of you this means the exam is due at 1:30pm Mountain Time.
- You can use the book, our class notes, your notes, our homework solutions, a calculator, and the internet (google, wikipedia, etc). **You are not allowed to communicate with any person regarding the exam** — not in person, not by email, not by phone, and not by asking about questions on online chat forums, or on online websites designed to enable cheating.
- On the paper that you submit, write the following statement and then sign your name.

  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

$\boxed{1}$ State the orbit-stabilizer theorem, and use it to deduce the size of the group G of the rotational symmetries of the octahedron, drawn below. The octahedron has 6 vertices and 8 triangular faces. Explain your work.



$\boxed{2}$ The alternating group A_3 is a normal subgroup of the symmetric group S_3, and hence we can form the quotient group S_3/A_3. Write down the elements of S_3/A_3 and draw the Cayley table for the group S_3/A_3. Explain your work.

$\boxed{3}$ Let n and k be integers, with $1 \leq k < n$, and with k relatively prime to n. Prove that the map $\phi\colon \mathbb{Z}_n \to \mathbb{Z}_n$ defined by $\phi(i) = k \cdot i$ is an automorphism. Explain your work.

$\boxed{4}$ State Lagrange's Theorem. Use Lagrange's Theorem to prove that if G is any finite group and $g \in G$, then $g^{|G|} = id$. Explain your work.

$\boxed{5}$ A student attempted to prove the following claim. The claim is correct, but the attempted proof has one key mistake. First, explain what is wrong with the student's attempted proof (what is the assumption the student made that they are not allowed to make)? Second, give your own correct proof for the claim.

Claim. In a group G, we have $(g^n)^{-1} = (g^{-1})^n$ for all $g \in G$ and integers $n \geq 1$.

Attempted proof that has a key error. For any $g \in G$ and $n \geq 1$, we have

$$\begin{aligned} g^n(g^{-1})^n &= \underbrace{g \cdot g \cdot \ldots \cdot g}_{n \text{ times}} \cdot \underbrace{g^{-1} \cdot g^{-1} \cdot \ldots \cdot g^{-1}}_{n \text{ times}} \\ &= \underbrace{(gg^{-1}) \cdot (gg^{-1}) \cdot \ldots \cdot (gg^{-1})}_{n \text{ times}} \\ &= \underbrace{id \cdot id \cdot \ldots \cdot id}_{n \text{ times}} \\ &= id. \end{aligned}$$

By a similar process, one can show $(g^{-1})^n g^n = id$. Hence $(g^n)^{-1} = (g^{-1})^n$. $\square$

This page intentionally left blank.

This page intentionally left blank.