CSU Math 366

Homework 9

Due Friday, April 17, at 5pm Mountain Time on Gradescope

Reading. Chapter 9

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Let H be a subgroup of group G, and let $a \in G$.
   (i) Give an example to show that it is possible to have $aH \neq Ha$, i.e., the left coset of H in G by a may not be equal to the right coset of H in G by a (write out the details of this example).
   (ii) Prove that if G is abelian, then necessarily $aH = Ha$.
2. Let H be a subgroup of group G, and let $a, b \in G$.
   (i) Prove that $a \in aH$.
   (ii) Prove that $(ab)H = a(bH)$.
   (iii) Prove that $aH = bH$ if and only if $a \in bH$.
   Hint: For $\Longleftarrow$*, use (ii).*
   (iv) For any $a, b \in G$, prove that either $aH = bH$ or else $aH \cap bH = \emptyset$.
   Hint: It suffices to show that if there is some $c \in aH \cap bH$*, then* $aH = bH$*.*
3. Let H be a subgroup of group G, and let $a, b \in G$. Prove that $|aH| = |bH|$.

   Hint: Define a function from aH *to* bH*, and show that this function is a bijection (both surjective and injective).*
4. Let G be a group of permutations of the set S, and let $i \in S$. Recall that the *stabilizer of i in G* is $\mathrm{Stab}_G(i) = \{\phi \in G \mid \phi(i) = i\}$. Prove that $\mathrm{Stab}_G(i)$ is a subgroup of G.