CSU Math 366

Homework 8

Due Friday, April 10, at 5pm Mountain Time on Gradescope

Reading. Chapter 7

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Let G and $\overline{G}$ be groups. Prove that if $\phi\colon G \to \overline{G}$ is an isomorphism, then so is $\phi^{-1}\colon \overline{G} \to G$.

   Remark: You may take it as a fact that if ϕ is bijective, then ϕ^{-1} is also bijective.

2. Let G be a group. Prove that if $\phi\colon G \to G$ and $\alpha\colon G \to G$ are automorphisms, then so is $\alpha \circ \phi\colon G \to G$.

   Remark: You may take it as a fact that $\alpha \circ \phi$ is bijective since both ϕ and α are.

3. Let G be a group. Prove that the set $\mathrm{Aut}(G)$ of automorphisms of G is also a group, with binary operation given by function composition.

   Remark: Write that #2 shows that composition is indeed a binary operation on $\mathrm{Aut}(G)$. Show that an identity exists. Use #1 to show that inverses exist. Say why we have associativity.

4. True or False. For the answers that are true, give a brief explanation of why. This could be no more than citing something from the book or from class. For the answers that are false, briefly say why (perhaps by giving a counterexample).

   (a) If G is a cyclic group, then $\mathrm{Aut}(G)$ is a cyclic group.

   (b) The groups D_6 and S_4 are isomorphic.

   (c) If groups G and $\overline{G}$ are isomorphic, then any bijective function $\phi\colon G \to \overline{G}$ taking the identity id_G in G to the identity $id_{\overline{G}}$ in $\overline{G}$ is an isomorphism.