CSU Math 366

Homework 7

Due Friday, April 3 at 5pm Mountain Time on Gradescope

Reading. Chapter 6

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Let $n \geq 2$. Prove that A_n is a subgroup of S_n by using the Two-Step Subgroup Test.

2. (a) Let $(\mathbb{R}, +)$ be the group of real numbers equipped with addition, and let $(\mathbb{R}_{>0}, \cdot)$ be the group of *positve* (that's what the > 0 means) real numbers equipped with multiplication. You may take it as a fact that the function $\phi\colon \mathbb{R} \to \mathbb{R}_{>0}$ defined by $\phi(x) = 2^x$ is bijective. Prove that ϕ is an isomorphism.

   (b) You may take it as a fact that the function $\phi\colon \mathbb{Z}_4 \to \{R_0, R_{90}, R_{180}, R_{270}\}$ defined by $\phi(j) = R_{90 \cdot j}$ is bijective. Prove that ϕ is an isomorphism.

   Remark: For your own education, it is a good idea to learn how to show that the function ϕ in (a) and the function ϕ in (b) are bijective. Please feel free for ask me for help on this!

3. The function $\phi\colon U(8) \to \mathbb{Z}_2 \times \mathbb{Z}_2$ defined by $\phi(1) = (0, 1)$, $\phi(3) = (0, 0)$, $\phi(5) = (1, 0)$, and $\phi(7) = (1, 1)$ is bijective. Nevertheless, show that ϕ is not an isomorphism.

   Hint: You should do this by finding specific elements $a, b \in U(8)$ such that $\phi(ab) \neq \phi(a) + \phi(b)$.

4. True or False. No justification is necessary to get credit (but you *ought* to know how to justify your answers; feel free to ask me for help on this.)

   (a) The function $\phi\colon \mathbb{R} \to \mathbb{R}$ defined by $\phi(x) = x^3$ is bijective.

   (b) The function $\phi\colon (\mathbb{R}, +) \to (\mathbb{R}, +)$ defined by $\phi(x) = x^3$ is an isomorphism.

   (c) If $\phi\colon G \to H$ is an isomorphism, then ϕ must map the identity in group G to the identity in group H.

   (d) If $\phi\colon G \to H$ is an isomorphism, then $\phi^{-1}\colon H \to G$ is an isomorphism.

   (e) If G and H are both groups of size 10, then G and H are isomorphic to each other.