CSU Math 366

Homework 5

Due Friday, February 28 at the beginning of class

Reading. Chapter 5

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. (a) Prove that if G is a group and $g \in G$ satisfies $g^n = e$, then $(g^{-1})^n = e$.
   Remark: One way to do this is to use a problem from HW3.

   (b) Prove that $|g| = |g^{-1}|$.
   Remark: Your proof could be organized as follows. "We will show $|g| \geq |g^{-1}|$ *and* $|g| \leq |g^{-1}|$*; together these imply* $|g| = |g^{-1}|$*. To see that* $|g| \geq |g^{-1}|$*, note that this follows from (a). To see that* $|g| \leq |g^{-1}|$*, . . . [DO SOME WORK HERE] . . . . Hence we are done."*

2. Write down the complete list of subgroups of $\mathbb{Z}_{24}$, sorted from the subgroup with the smallest order to the subgroup with the largest order. Each subgroup should appear once and only once. List the order of each subgroup, and for each subgroup give an element that generates that subgroup.

3. The *center* $Z(G)$ of a group G is the subset of the elements that commute with all elements of G. That is,

   $$Z(G) = \{a \in G \mid ax = xa \text{ for all } x \in G\}.$$

   Use the Two-Step Subgroup Test to show that $Z(G)$ is a subgroup of G.

4. Write the element $(1246)(45)(263)(1892673)(89) \in S_9$ in disjoint cycle form.