

Homework 4

Due Friday, February 21 at the beginning of class

Reading. Chapter 4

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Prove that if G is a group and a is an element in G , then a has a unique inverse in G .

Remark: Use multiplicative notation.

2. Re-write the above proof in additive notation, following exactly the same steps you did above.

Remark: 0 is a good name for the identity when using additive notation. Additive notation is typically used only when the group G is commutative, but don't assume that G is commutative here — it's not needed.

3. Prove that if $\gcd(k, n) = 1$, then $k \in \mathbb{Z}_n$ generates \mathbb{Z}_n .

Remark: You can't cite Corollary 4 on page 80 (or Corollary 3 on page 80); I am asking you to reprove one direction of this result. You should refer to our notes from class!

4. Use the (extended) Euclidean Algorithm to find integers $s, t \in \mathbb{Z}$ such that $51s + 187t = \gcd(51, 187)$.

Remark: You can do the computations with no words at all, but then at the end you should conclude by writing "So $s = ??$ and $t = ??$ solves $51s + 187t = \gcd(51, 187)$."