CSU Math 366

# Homework 1

Due Friday, January 31 at the beginning of class

**Reading.** Chapters 1, 2

**Remark.** Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words. For example, suppose the assigned problem were "Solve $x^2-3x+2=0$." The answer

$$\text{“}x^2-3x+2=0 \qquad (x-1)(x-2)=0 \qquad x=1 \text{ or } x=2,\text{”}$$

would not make me 100% happy, but the following answer would:

$$\text{“Since } x^2-3x+2=0 \text{ implies } (x-1)(x-2)=0, \text{ we have } x=1 \text{ or } x=2.\text{”}$$

Note we added only four English words.

*For every homework in this class, a homework problem with no English words will be returned without being graded. For #4 on this homework, for example, you could write "Below is a multiplication table for $D_5$," and that would be more than sufficient.*

Let $D_5=\{R_0,R_{72},R_{144},R_{216},R_{288},F_1,F_2,F_3,F_4,F_5\}$ be the group of symmetries of the regular pentagon, or in other words, the *dihedral group of order 10*. The elements of this group are drawn below. The five rotations $R_0$, $R_{72}$, $R_{144}$, $R_{216}$, $R_{288}$ are counterclockwise rotations by 0°, 72°, 144°, 216°, 288°. The five flips $F_1$, $F_2$, $F_3$, $F_4$, $F_5$ are through the vertices labeled $1,2,3,4,5$ in counterclockwise order, with vertex 1 at the top, as drawn below.

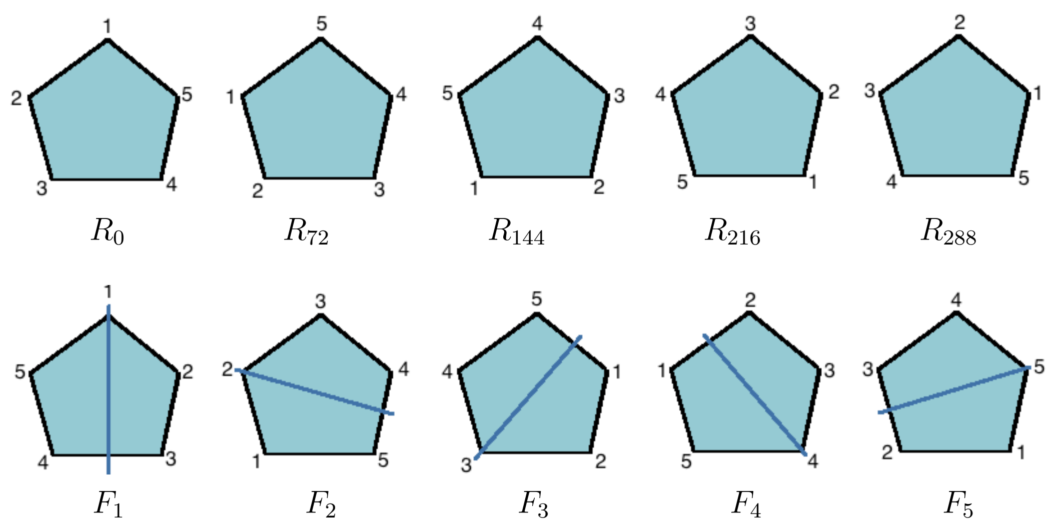

Image credit: http://mathonline.wikidot.com/the-group-of-symmetries-of-the-pentagon

**Problems.**

1. Draw a picture showing why $F_5F_3 = R_{288}$.

   *Remark: This picture could be analogous to the picture why, in our class notes ([1]) on page 2, we have that $HR_{90} = D$ in the group $D_4$.*

2. What is $F_3R_{288}$? What is $R_{288}F_3$?
   Since it is not the case that $ba = ab$ for all elements $a, b \in D_5$, this means that the group $D_5$ is not *commutative*, or equivalently, not *Abelian* (these two words mean the same thing).

3. Verify that $F_2(F_3R_{144}) = (F_2F_3)R_{144}$.
   It turns out that $c(ba) = (cb)a$ for all $a, b, c \in D_5$, and for this reason we say that $D_5$ is *associative*.

4. Fill out the multiplication table (or Cayley table) for the dihedral group $D_5$ of symmetries of the regular pentagon. As we did in class for $D_4$, write the composition $ba$ (which means "do $a$ first and $b$ second") in the column corresponding to $a \in D_5$ and in the row corresponding to $b \in D_5$.

First operation

| | $R_0$ | $R_{72}$ | $R_{144}$ | $R_{216}$ | $R_{288}$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | | | | | | | | | | |
| $R_{72}$ | | | | | | | | | | |
| $R_{144}$ | | | | | | | | | | |
| $R_{216}$ | | | | | | | | | | |
| $R_{288}$ | | | | | | | | | | |
| $F_1$ | | | | | | | | | | |
| $F_2$ | | | | | | | | | | |
| $F_3$ | | | | | | | | | | |
| $F_4$ | | | | | | | | | | |
| $F_5$ | | | | | | | | | | |

$\nwarrow$ Second operation

[1] https://www.math.colostate.edu/~adams/teaching/Notes_Math366.pdf