**CSU Math 366** **Practice Midterm 1**

Name: _____

- For #1, #2, #3, and #4, explain your logic fully and write complete sentences. For #5, just say "True" or "False". No partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement: "I will not give, receive, or use any unauthorized assistance."

Signature: _____

| Problem | Total Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 50 | |

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1 (a) (5 points) Define what it means for a group $G$ to be *commutative* (which means the same thing as *Abelian*).

(b) (5 points) Let $G$ be a group and let $g \in G$. Define the *order* of $g$ (denoted $|g|$).

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2 (a) (5 points) Draw the Cayley table (multiplication table) for $U(8)$. Is $U(8)$ a cyclic group?

*Remark: Recall that $U(8)$ is the set of numbers less than 8 that are relatively prime to 8, with group operation multiplication modulo 8.*

(b) (5 points) Write $(123)(124)(24) \in S_4$ in disjoint cycle form.

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3 Let $G$ be a group, and let $a \in G$.

(a) (3 points) Define $\langle a \rangle$, the cyclic subgroup generated by $a$.

(b) (7 points) Prove that $\langle a \rangle$ is a subgroup of $G$ by using the Two-Step Subgroup Test.

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4 Let $G$ be a group. Prove the cancellation law. That is, prove that if $a, b, c \in G$ satisfy $ab = ac$, then this implies that $b = c$.

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5 No justification needed: just say "True" or "False". No partial credit.

(a) True or False: Let $G$ be a group that is commutative. If $ab = ca$ for elements $a, b, c \in G$, then this implies that $b = c$.

(b) True or False: The element 17 generates $\mathbb{Z}_{99}$.

(c) True or False: Let $S$ be a set, and let $\star$: $S \times S \to S$ be a binary relation. If $\star$ is commutative, then $\star$ is also associative.

(d) True or False: If $G$ is a group and $g \in G$ satisfies $|g| = 10$, then $g^{25} \neq id$.

(e) True or False: The subset $\{R_0, H, V, D, D'\}$ forms a subgroup of $D_4 = \{R_0, F_{90}, R_{180}, R_{270}, H, V, D, D'\}$, the symmetries of the square under composition.

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