CSU Math 366
Spring 2019

Homework 5

Due Friday, March 1 at the beginning of class

Reading. Chapter 5

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Prove that if G is a group and a is an element in G, then a has a unique inverse in G.

   Remark: Use multiplicative notation.

2. Re-write the above proof in additive notation, following exactly the same steps you did above.

   Remark: 0 *is a good name for the identity when using additive notation. Additive notation is typically used only when the group G is commutative, but don't assume that G is commutative here — it's not needed.*

3. (a) Prove that if G is a group and $g \in G$ satisfies $g^n = e$, then $(g^{-1})^n = e$.

   Remark: One way to do this is to use #1 and a problem from HW3.

   (b) Prove that $|g| = |g^{-1}|$.

   Remark: Your proof could be organized as follows. "We will show $|g| \geq |g^{-1}|$ and $|g| \leq |g^{-1}|$; together these imply $|g| = |g^{-1}|$. To see that $|g| \geq |g^{-1}|$, note that this follows from (a). To see that $|g| \leq |g^{-1}|$, ... [DO SOME WORK HERE] .... Hence we are done."

4. Write down the complete list of subgroups of $\mathbb{Z}_{12}$, sorted from the subgroup with the smallest order to the subgroup with the largest order. Each subgroup should appear once and only once. List the order of each subgroup, and for each subgroup give an element that generates that subgroup.