

Homework 4

Due Friday, February 22 at the beginning of class

Reading. Chapter 4

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Prove that the identity element in a group G is unique.
2. Prove that if $\gcd(k, n) = 1$, then $k \in \mathbb{Z}_n$ generates \mathbb{Z}_n .

Remark: You can't cite Corollary 4 on page 80; I am asking you to reprove one direction of this result. You should refer to our notes from class!

3. The *center* $Z(G)$ of a group G is the subset of the elements that commute with all elements of G . That is,

$$Z(G) = \{a \in G \mid ax = xa \text{ for all } x \in G\}.$$

Use the Two-Step Subgroup Test to show that $Z(G)$ is a subgroup of G .

4. Use the (extended) Euclidean Algorithm to find integers $s, t \in \mathbb{Z}$ such that $51s + 187t = \gcd(51, 187)$.

Remark: You can do the computations with no words at all, but then at the end you should conclude by writing "So $s = ??$ and $t = ??$ solves $51s + 187t = \gcd(51, 187)$."