CSU Math 366
Spring 2019

Homework 2

Due Friday, February 8 at the beginning of class

Reading. Chapter 3

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. In this exercise we will check that it is possible for a binary operation to be commutative but not associative. Define the binary operation $\diamond\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ on the real numbers by $x \diamond y = xy + 1$. Since $x \diamond y = xy + 1 = yx + 1 = y \diamond x$, we see that $\diamond$ is commutative. Show that $\diamond$ is not associative, as follows:

   - Compute a formula for $(x \diamond y) \diamond z$, and compute a formula for $x \diamond (y \diamond z)$.
   - Say why it is sometimes possible to have $(x \diamond y) \diamond z \neq x \diamond (y \diamond z)$; for example you could pick specific choices of $x, y, z \in \mathbb{R}$ to show this.

   Remark: Other binary operations on $\mathbb{R}$ *that are commutative but not associative include* $x \diamond y = \frac{x+y}{2}$ *or* $x \diamond y = |x - y|$.

2. Recall from class or our book that $\mathbb{GL}(2, \mathbb{R})$ is the group of all 2×2 matrices with non-zero determinant, under the group operation of matrix multiplication. Show that $\mathbb{GL}(2, \mathbb{R})$ is not commutative.

3. Exercise 22 from our book: Let G be a group with the property that for any $x, y, z \in G$, we have that $xy = zx$ implies $y = z$. Prove that G is commutative (that is, that for all $a, b \in G$, we have $ab = ba$).

   Hint: The first sentence of your proof should be something like "Let a *and* b *be arbitrary elements of* G*." The last sentence of your proof should be something like "Since* $ab = ba$*, we have shown that* G *is commutative."*

   Hint: Since we are trying to show $ab = ba$*, what is a reasonable choice for* y *and for* z*? Then, once you have a good choice in mind for* y *and for* z*, is there a choice of* x *that makes* $xy = zx$ *true?*

4. List all the elements of $U(12)$, write down the Cayley table (multiplication table) for this group, say what the identity in this group is, and say which elements are inverses in this group.