CSU Math 366

Homework 10

Due Friday, April 26 at the beginning of class

Reading. Chapter 10

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Let K be the subgroup of D_4 given by $K = \{R_0, R_{180}\}$. Prove that K is normal in D_4 by showing that $xK = Kx$ for all $x \in D_4$. Draw the Cayley table for the quotient group D_4/K.

2. Let H be a subgroup of G with $|G|/|H| = 2$. Prove that H is normal in G.

   Hint: Let $x \in G$. If $x \notin H$, then explain why xH is the set of all elements in G not in H. Is the same true for Hx?

   Remark: The above problem shows that A_n is a normal subgroup of the symmetric group S_n, since $|S_n|/|A_n| = 2$. It also shows that the subgroup Rot of all rotations is a normal subgroup of the dihedral group D_n, since $|D_n|/|Rot| = 2$.

3. Let H be a subgroup of group G. Prove that H is a normal subgroup of G if and only if $xHx^{-1} \subseteq H$ for all $x \in G$.

4. Let $\phi\colon G \to \overline{G}$ be a *homomorphism*, which means that for all $a, b \in G$ we have $\phi(ab) = \phi(a)\phi(b)$. We will study homomorphisms in Chapter 10; if a homomorphism is bijective then it is called an *isomorphism* (which we have seen before).

   Given a homomorphism $\phi\colon G \to \overline{G}$, its *kernel* is defined as

   $$\ker \phi = \{x \in G \mid \phi(x) = id_{\overline{G}}.\}$$

   Here $id_{\overline{G}}$ is the identity in $\overline{G}$. Use the Two-Step Subgroup Test to show that $\ker \phi$ is a subgroup of G.