CSU Math 366
Spring 2019

Homework 1

Due Friday, February 1 at the beginning of class

Reading. Chapters 1, 2

Let $D_5 = \{R_0, R_{72}, R_{144}, R_{216}, R_{288}, F_1, F_2, F_3, F_4, F_5\}$ be the group of symmetries of the regular pentagon, or in other words, the *dihedral group of order 10*. The elements of this group are drawn below. The five rotations R_0, R_{72}, R_{144}, R_{216}, R_{288} are counterclockwise rotations by 0°, 72°, 144°, 216°, 288°. The five flips F_1, F_2, F_3, F_4, F_5 are through the vertices labeled $1, 2, 3, 4, 5$ in counterclockwise order, with vertex 1 at the top, as drawn below.

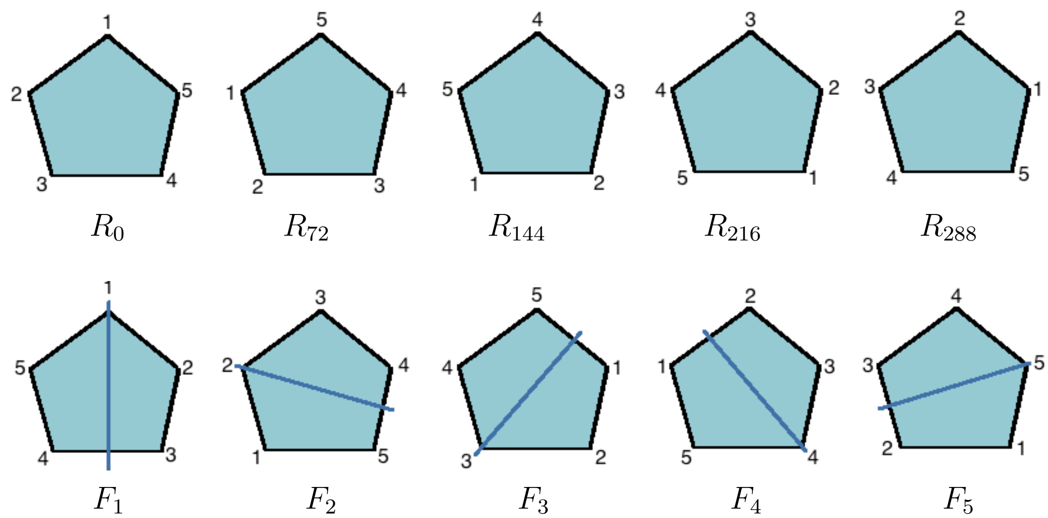

Image credit: http://mathonline.wikidot.com/the-group-of-symmetries-of-the-pentagon

Problems.

1. Draw a picture showing why $F_5F_3 = R_{288}$.

   Remark: This picture could be analogous to the picture why, in our class notes ([1]) *on page 2, we have that* $HR_{90} = D$ *in the group* D_4.

2. What is F_2R_{144}? What is $R_{144}F_2$?
   Since it is not the case that $ba = ab$ for all elements $a, b \in D_5$, this means that the group D_5 is not *commutative*, or equivalently, not *Abelian* (these two words mean the same thing).

[1] https://www.math.colostate.edu//~adams/teaching/math366spr2019/NotesMath366.pdf

3. Verify that $F_4(R_{72}F_1) = (F_4R_{72})F_1$.
   It turns out that $c(ba) = (cb)a$ for all $a, b, c \in D_5$, and for this reason we say that D_5 is *associative*.

4. Fill out the multiplication table (or Cayley table) for the dihedral group D_5 of symmetries of the regular pentagon. As we did in class for D_4, write the composition ba (which means "do a first and b second") in the column corresponding to $a \in D_5$ and in the row corresponding to $b \in D_5$.

First operation

	R_0	R_{72}	R_{144}	R_{216}	R_{288}	F_1	F_2	F_3	F_4	F_5
R_0										
R_{72}										
R_{144}										
R_{216}										
R_{288}										
F_1										
F_2										
F_3										
F_4										
F_5										

$\nwarrow$ Second operation