

## Homework 9

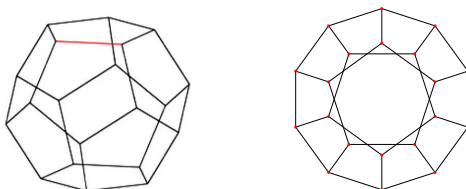
Due Friday, November 15 at the beginning of class

**Reading.** Sections 8.1, 8.2, 8.3

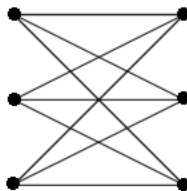
**Remark.** Make grammatically correct sentences by adding in just a few English words.

**Problems.**

1. Suppose we have a polyhedron that is a triangulation of the sphere with 45 vertices and 90 edges. How many faces must it have?
2. Is it possible to find a set of 20 edges from  $K_{10}$ , the complete graph on 10 vertices, such that if you remove those 20 edges then you obtain a planar graph? If so, draw such a graph as a planar map (with no edges crossing). If not, explain why not.
3. Consider the “dodecahedron graph,” drawn in two different ways below. Is this graph planar? If so, draw it as a planar map (with no edges crossing). If not, explain why not.



4. Prove that the graph  $K_{3,3}$  drawn below is not planar. You are not allowed to cite Kuratowski's Theorem which I mentioned in class — I am asking you to prove a special case of this theorem.



*Hint 1: Observe that there are no triangles (cycles of length 3) in the graph. Hence we can mimic the proof of Theorem 12.2.2, except now each face has at least 4 edges on its boundary (since there are no triangles).*

*Hint 2: If you get stuck, see Exercise 12.2.2 and its solution in our book.*