## Homework 8

## Due Friday, November 1 at the beginning of class

## Reading.

Sections 12.1, 12.2, 12.3
Remark. Make grammatically correct sentences by adding in just a few English words.

## Problems.

1. In how many ways can you cover a $3 \times 12$ chessboard with identical dominoes, where you must use exactly 12 dominoes each of size $3 \times 1$ ? Fully justify your answer.
2. Use the Euclidean algorithm (show your work) to find an integer $x$ between 0 and 70 such that $11 x+2 \equiv 12 \bmod 71$.
3. (a) Does there exist a graph with 8 vertices of degrees 1,2,2,3,3,4,4,4? Explain.
(b) Does there exist a graph with 8 vertices of degrees $0,1,1,2,2,3,4,7$ ? Explain.
(c) Does there exist a graph with 8 vertices of degrees $2,2,2,2,2,3,3,6$ ? Explain.
4. How many subgraphs does a cycle of length 4 have? Assume the four vertices are labelled $a, b, c, d$. We consider the subgraph with two vertices $a, b$ and a single edge $\{a, b\}$ to be different from the subgraph with two vertices $b, c$ and a single edge $\{b, c\}$. Remark: The graph with no vertices and no edges is a subgraph of every graph. Also, a graph is always a subgraph of itself. A subgraph could be disconnected; for example we could have a subgraph with three vertices $a, b, d$ and a single edge $\{a, b\}$. I suggest you count the number of subgraphs with 0 vertices, then the number of subgraphs with 1 vertex, then the number of subgraphs with 2 vertices, ..., etc.
