Homework 8

Due Friday, November 1 at the beginning of class

Reading.

Sections 12.1, 12.2, 12.3

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

- 1. In how many ways can you cover a 3×12 chessboard with identical dominoes, where you must use exactly 12 dominoes each of size 3×1 ? Fully justify your answer.
- 2. Use the Euclidean algorithm (show your work) to find an integer x between 0 and 70 such that $11x + 2 \equiv 12 \mod 71$.
- 3. (a) Does there exist a graph with 8 vertices of degrees 1,2,2,3,3,4,4,4? Explain.
 - (b) Does there exist a graph with 8 vertices of degrees 0,1,1,2,2,3,4,7? Explain.
 - (c) Does there exist a graph with 8 vertices of degrees 2,2,2,2,3,3,6? Explain.
- 4. How many subgraphs does a cycle of length 4 have? Assume the four vertices are labelled a, b, c, d. We consider the subgraph with two vertices a, b and a single edge $\{a, b\}$ to be different from the subgraph with two vertices b, c and a single edge $\{b, c\}$.

Remark: The graph with no vertices and no edges is a subgraph of every graph. Also, a graph is always a subgraph of itself. A subgraph could be disconnected; for example we could have a subgraph with three vertices a, b, d and a single edge $\{a, b\}$. I suggest you count the number of subgraphs with 0 vertices, then the number of subgraphs with 1 vertex, then the number of subgraphs with 2 vertices, ..., etc.