

Homework 6

Due Friday, October 18 at the beginning of class

Reading.

Sections 6.6, 6.7, 6.8, 6.9

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. In how many ways can people (any number of people) sit in a row of 9 subway seats, so that no two people sit next to each other?

For example, there are two ways for people to sit in 1 seat: it could be empty or full.

For example, there are three ways for people to sit in 2 seats: “empty empty”, “empty full”, or “full empty”. The configuration “full full” is not possible since no two people are allowed to sit next to each other.

Hint: Write “Let S_n be the number of ways for people to sit in a row of n seats.” Find and justify a recurrence relation for S_n .

2. All variables in this problem are integers. Show that

(a) If $a \mid b$ and $b \mid c$ then $a \mid c$.

(b) If $a \mid b$ and $a \mid c$ then $a \mid (b - c)$.

(c) If $a \mid b$ and $a \nmid c$ then $a \nmid (b - c)$.

Hint: Suppose for a contradiction that we had $a \mid (b - c)$. Use $a \mid b$ to show this would imply $a \mid c$, a contradiction.

(d) If p is a prime and $p \mid ab$, then either $p \mid a$ or $p \mid b$ (or both).

Hint: Consider prime factorizations!

3. Prove that if p is a prime number, then \sqrt{p} is irrational.

Hint: Edit, as needed, our proof from class that $\sqrt{2}$ is irrational

4. A deck of cards consists of $52 = 13 \cdot 4$ cards: an ace, two, three, \dots , nine, ten, jack, queen, and king (13 values) from each of 4 different suits: $\heartsuit, \diamondsuit, \clubsuit, \spadesuit$. A poker hand consists of a subset of 5 of these 52 cards.

(a) How many 4-of-a-kind poker hands are there? We consider the hands $\{K\heartsuit, K\diamondsuit, K\spadesuit, K\clubsuit, 2\diamondsuit\}$ and $\{2\diamondsuit, K\spadesuit, K\clubsuit, K\diamondsuit, K\heartsuit\}$ to be the same.

(b) What fraction of all possible poker hands are 4-of-a-kinds?