CSU Math 301

Homework 4

Due Friday, September 27 at the beginning of class

Reading. Sections 3.5, 3.6, 4.1, 4.2

Remark. Make grammatically correct sentences by adding in just a few English words (for problems 1-3, but not for problem 4).

Problems.

1. Prove by induction that $3^n \geq 5n + 10$ for all $n \geq 3$.

2. You want to buy a bag of 13 marbles, and there are 4 different marble colors (red, blue, green, yellow) to choose from. Any two marbles of the same color are indistinguishable. Buying 10 red and 3 blue marbles gives the same bag as buying 3 blue and 10 red marbles. Also, a bag consisting of 13 green marbles (and no other colors) is certainly possible. How many different bags could you buy?

3. (a) How many different anagrams (i.e. strings of length 12) can you form by rearranging the letters of the word COLLYWOBBLES? For example, WOLLYCOBBLES and LLLBBOOCYWES are two such rearrangements.

   (b) How many ways are there to place 8 rooks on a chessboard with no two attacking each other if 3 are wooden, 3 are marble, and 2 are plastic? Rooks made from the same material are indistinguishable.

4. This last question is a "short answer" question, meaning no English words or explanations are required. Simply write down the correct mathematical expression.

   How may ways are there to distribute 15 toys to 5 (distinguishable) people A, B, C, D, E if ...

   (a) ... the toys are identical?

   (b) ... the toys are identical and every person must get at least one?

   (c) ... the toys are distinguishable and we give 4 toys to A, 4 toys to B, 3 toys to person C, 2 toys to D, and 2 toys to E?

   (d) ... the toys are distinguishable and it is not required that every person get one? For example, we could give all 15 toys to person A.