

Homework 3

Due Monday, September 23 at the beginning of class

Reading.

Sections 3.1, 3.2, 3.3, 3.4

Remark. Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words.

Problems.

1. Prove by induction that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ for all $n \geq 1$. (There are also other ways to prove this statement, but I want you to give a proof by induction).
2. Prove that $n! \geq 2^n$ for all $n \geq 4$.
3. (a) If 11 numbers are chosen from 1 to 100, then show that there are two of them whose difference is less than 10.
(b) If 7 numbers are chosen from 1 to 100, then show that there are two of them whose difference is less than 17.
4. (a) If 95 people are seated in a row of 100 chairs, then show that some consecutive set of 16 chairs are filled with people.
(b) If 95 people are seated in a *circle* of 100 chairs, then show that some consecutive set of 19 chairs are filled with people.