CSU Math 301

Homework 2

Due Friday, September 13 at the beginning of class

Reading. Sections 1.8, 2.1, 2.4

Remark. Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words.

Problems.

1. You have eight different dolls and your sibling has ten different baseball cards. In how many ways could you trade three of your eight dolls for four of your sibling's baseball cards?

2. Let A and B be sets with sizes $|A| = 9$ and $|B| = 4$.

   (a) What are all the possible values of $|A \cap B|$?

   (b) What are all the possible values of $|A \cup B|$?

   (c) What are all the possible values of $|B \setminus A|$? [See the definition of $B \setminus A$ in Section 1.2 of our class notes.]

3. (a) Suppose $\binom{n}{k-1} = 210$ and $\binom{n+1}{k} = 462$. What is $\binom{n}{k}$?
   Hint: The best way to solve this problem does not require you to find n or k.

   (b) Give an algebraic proof that $\binom{n}{2} + \binom{n+1}{2} = n^2$.
   Remark. I do *not* suggest using a proof by induction.

4. You have a large supply of each of 3 different kinds of postcards (a ram, buffalo, and goat print postcard). You want to send postcards to 11 different friends.

   (a) How many ways are there to send postcards if to each friend you want to send exactly one postcard? If Alice gets a ram postcard and Bob gets a buffalo postcard, that's different than if Alice gets a buffalo postcard Bob gets a ram postcard.

   (b) How many ways are there to send postcards if to each friend you want to send either 1, 2, or 3 postcards, such that no friend gets two postcards of the same animal? For example, you may decide to send Alice the ram, buffalo, and goat postcards, while to Bob you send only the ram postcard. You may certainly choose to send Alice the same nonempty subset that you choose to send to Bob.