

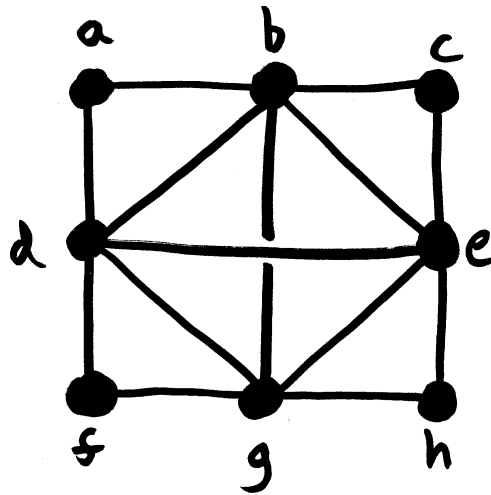
## *Practice* Homework 12

Due never!

**Remark.** Make grammatically correct sentences by adding in just a few English words.

## Problems.

1. Show that the graph drawn below is not 3-colorable but is 4-colorable.



2. Let  $G$  be a connected graph such that all of the vertices except  $d + 1$  of the vertices have degree at most  $d$  (the remaining  $d + 1$  vertices may have degree larger than  $d$ ). Prove that  $G$  is  $(d + 1)$ -colorable.
3. Let  $G$  be a connected weighted graph with positive edge costs.
  - (a) Describe how to find a spanning tree for which the sum of the edge-costs is *maximal*.  
*Hint: Create a new weighted graph  $G'$  by multiplying each edge weight of  $G$  by  $-1$ .*
  - (b) Describe how to find a spanning tree for which the *product* of the edge-costs is minimal.  
*Hint: Create a new weighted graph  $G'$  by editing the edge weights of  $G$  using logarithms.*
4. Show by an example that if we don't assume the triangle inequality, then a tour found by the Tree Shortcut Algorithm can be longer than 1000 times an optimal tour.