CSU Math301

## Homework 11

Due Friday, December 6 at the beginning of class

Reading. Sections 13.1, 13.2, 13.3, 13.4

Remark. Make grammatically correct sentences by adding in just a few English words.

## Problems.

- 1. Show that a graph G is a tree if and only if it contains no cycles, but adding any new edge creates a cycle. This is part (b) of Theorem 8.1.1.
- 2. Is the given code a valid Prüfer code? Explain why or why not *prior* to computing the extended Prüfer code. If the code is not valid, then you are done! If the code is valid, then compute the extended Prüfer code and draw the corresponding tree.
  - (a) 4205423
  - (b) 1592016
  - (c) 444444
- 3. (a) Find a minimal spanning tree on the weighted graph drawn below. What is the name of the algorithm you used?



(b) Draw a connected weighted graph that has at least two different minimal spanning trees.

- (c) Draw a connected weighted graph which has a unique minimal spanning tree, but in which at least two edges have the same weight.
- 4. Let G be a connected weighted graph in which all edge weights are positive. Show that the cost of a minimal spanning tree is *strictly* smaller (< instead of  $\leq$ ) than the cost of an optimal tour (i.e., a tour solving the Traveling Salesperson Problem).