

## Homework 1

Due Friday, September 6 at the beginning of class

### Reading.

Sections 1.3, 1.5, 1.6, 1.7 (we're skipping Section 1.4)

**Remark.** Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words. For example, suppose the assigned problem were “Solve  $x^2 - 3x + 2 = 0$ .” The answer

$$x^2 - 3x + 2 = 0 \quad (x - 1)(x - 2) = 0 \quad x = 1 \text{ or } x = 2,$$

would not make me 100% happy, but the following answer would:

“Since  $x^2 - 3x + 2 = 0$  implies  $(x - 1)(x - 2) = 0$ , we have  $x = 1$  or  $x = 2$ .”

Note we added only four English words. As a second example, suppose the problem were “How many ways can 47 students sit in 47 seats?” I would not be 100% happy with the answer “47!”, but I would be happy with “Since there are 47 ways to choose the student in the first seat, 46 ways to choose the student in the second seat, etc, there are 47! ways.”

### Problems.

1. Let  $A$  and  $B$  be sets. What is  $|A \cup B|$  if  $|A| = 15$ ,  $|B| = 9$ , and  $|A \cap B| = 5$ ?
2. How many ways are there to group 16 people into eight pairs of two?
3. In how many ways can you visit 5 of the 50 state capitals? The trip

Sacramento  $\rightarrow$  Dover  $\rightarrow$  Baton Rouge  $\rightarrow$  Bismarck  $\rightarrow$  Denver

is certainly different than the trip

Dover  $\rightarrow$  Baton Rouge  $\rightarrow$  Denver  $\rightarrow$  Bismarck  $\rightarrow$  Sacramento.

4. (a) Find the number of all 5-digit strings of letters. String  $abcde$  is different from  $acbde$ .  
 (b) Find the number of all 5-digit strings of letters in which no two consecutive letters are the same. For example, strings  $xdwax$  and  $xdxdg$  count but strings  $xdwwx$  and  $xddxg$  do not.  
 (c) Find the number of all 5-digit strings of letters in which letters which differ by one slot are not the same, and also letters which differ by two slots are not the same. For example, string  $xwzzy$  does not count since there are  $z$ 's which differ by one slot, string  $xzwzy$  does not count since there are  $z$ 's which differ by two slots, but string  $zxwzy$  is okay.