

Homework 9

Due Friday, October 26 at the beginning of class

Reading.

Sections 12.1, 12.2, 12.3

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Solve $9x \equiv 4 \pmod{97}$ for some integer $x \in \mathbb{Z}$ between 0 and 96.
(We're looking for $x \in \mathbb{Z}$, so $x = \frac{4}{9}$ is incorrect.)
2. How many subgraphs does a cycle of length 4 have? Assume the four vertices are labelled a, b, c, d . We consider the subgraph with two vertices a, b and a single edge $\{a, b\}$ to be different from the subgraph with two vertices b, c and a single edge $\{b, c\}$.
Remark: The graph with no vertices and no edges is a subgraph of every graph. Also, a graph is always a subgraph of itself. A subgraph could be disconnected; for example we could have a subgraph with three vertices a, b, d and a single edge $\{a, b\}$. I suggest you count the number of subgraphs with 0 vertices, then the number of subgraphs with 1 vertex, then the number of subgraphs with 2 vertices, ..., etc.
3. (a) Does there exist a graph with 7 vertices of degrees 0,2,2,2,4,4,6? Explain.
(b) Does there exist a graph with 7 vertices of degrees 2,2,3,3,4,4,5? Explain.
4. (a) Formulate the following assertion as a theorem about graphs: "At every party there are at least two people who know the same number of other people at the party". Your theorem should be of the following form: "In any graph G with at least 2 vertices, [some property about vertex degrees holds]".
(b) Give a proof/explanation of this theorem about graphs.