CSU Math 301
Fall 2018

Homework 7

Due Friday, October 12 at the beginning of class

Reading.
Sections 6.6, 6.7, 6.8, 6.9

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. (a) How many 4-of-a-kind poker hands are there? We consider the hands $\{K\heartsuit, K\diamondsuit, K\spadesuit, K\clubsuit, 2\diamondsuit\}$ and $\{2\diamondsuit, K\spadesuit, K\clubsuit, K\diamondsuit, K\heartsuit\}$ to be the same.

   (b) How many 3-of-a-kind poker hands are there?
   (Neither a full house nor a 4-of-a-kind are considered to be 3-of-a-kind hands).

2. In how many ways can you cover a $2 \times n$ chessboard with identical dominoes, where you must use exactly n dominoes each of size 2×1? Fully justify your answer.

   Hint: Write "Let S_n be the number of ways to cover a board of size $2 \times n$."

3. All variables in this problem are integers. Show that

   (a) If $a \mid b$ and $b \mid c$ then $a \mid c$.

   (b) If $a \mid b$ and $a \mid c$ then $a \mid (b + c)$.

   (c) If $a \mid b$ and $a \nmid c$ then $a \nmid (b + c)$.
   Hint: Suppose for a contradiction that we had $a \mid (b + c)$. Use $a \mid b$ to show this would imply $a \mid c$, a contradiction.

   (d) If p is a prime and $p \mid ab$, then either $p \mid a$ or $p \mid b$ (or both).
   Hint: Consider prime factorizations!

4. Prove that if n is a positive integer that is not a square (i.e. there is no integer m with $n = m^2$), then $\sqrt{n}$ is irrational.

   Hint: Edit our proof from class that $\sqrt{2}$ is irrational. If you get stuck, then see the hint in the back of the book for Exercise 6.3.6.