

Homework 6

Due Friday, October 5 at the beginning of class

Reading.

Sections 4.3, 6.1, 6.2, 6.3, 6.4

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

1. Define the *Lucas numbers* by $L_0 = 2$, $L_1 = 1$, and $L_{n+1} = L_n + L_{n-1}$ for $n \geq 1$. Find L_{10} .
2. When climbing a staircase, you can take either one, two, or three stairs in a single step.
 - (a) Write “Let S_n be the number of ways to climb a staircase with n stairs.” Prove the recurrence relation $S_n = S_{n-1} + S_{n-2} + S_{n-3}$.
 - (b) Write down what S_0 , S_1 , and S_2 are (note $S_0 = 1$ since there is one way to climb a staircase with zero stairs: do nothing). No justification needed.
 - (c) Use (a) and (b) to answer the following question. How many ways are there to climb a staircase with 9 stairs?
3. When climbing a staircase, you can take either one or three stairs in a single step. How many ways are there to climb a staircase with 9 stairs? Your answer should include analogous versions of (a), (b), and (c) from #1 above.
4. Let F_n be the n -th Fibonacci number. Prove that $F_1^2 + \dots + F_n^2 = F_n F_{n+1}$ for all $n \geq 1$.