

Homework 3

Due Friday, September 7 at the beginning of class

Reading. Sections 1.8, 2.1, 2.4

Remark. Your answers should be briefly explained. If you're only writing math symbols, then you're not explaining things — make grammatically correct sentences by adding in just a few English words.

Problems.

- Let A and B be sets with sizes $|A| = 4$ and $|B| = 7$.
 - What are all the possible values of $|A \cup B|$?
 - What are all the possible values of $|A \cap B|$?
 - What are all the possible values of $|B \setminus A|$?
- Alice has six wristbands of different colors and Bob has seven necklaces of different colors. In how many ways can Alice trade two of her wristbands for three of Bob's necklaces?
- Suppose $\binom{n}{k-1} = 120$ and $\binom{n+1}{k} = 330$. What is $\binom{n}{k}$?
Hint: The best way to solve this problem does not require you to find n or k .
 - Give an algebraic proof that $\binom{n}{2} + \binom{n+1}{2} = n^2$.
Remark. I do *not* suggest using a proof by induction.
- How many different strings of length 9 can you form by rearranging the letters of the word TENNESSEE? For example, SENESENET and SNEEETENS are two such rearrangements.
(This is a preview of something we will learn again in Section 3.3.)
- You have a large supply of each of 4 different kinds of postcards (a ram, buffalo, fox, and goat print postcard). You want to send postcards to 10 different friends.
 - How many ways are there to send postcards if to each friend you want to send exactly one postcard? If Alice gets a ram postcard and Bob gets a buffalo postcard, that's different than if Alice gets a buffalo postcard Bob gets a ram postcard.
 - How many ways are there to send postcards if to each friend you want to send either 1, 2, 3, or 4 postcards, such that no friend gets two postcards of the same animal? For example, you may decide to send Alice the ram, buffalo, and fox postcards, while to Bob you send only the ram postcard.